Building Math Vocabulary and Math Operations Recognition in a Montessori E1 Classroom

By

Karen G. Olson

A Master's Paper

Submitted in Partial Fulfillment of

The Requirements for the Degree of

Master of Science in Education—Montessori

_____

Major Advisor's Signature

_____

Date

University of Wisconsin-River Falls

2015

Abstract

This research explored methods to develop tools and systems that build math vocabulary and recognition of the type of math operations required according to problems posed in a lower elementary classroom. Participants in the research were members of a Lower Elementary classroom at a public Montessori elementary school in a small upper Midwestern city. The ages of the students ranged from age 6 to 9 years. The researcher, a Montessori teacher, developed tools and systems to build math vocabulary and recognition of math operations in an environment where the developing autonomy of students in completing learning tasks is valued. Data was collected through observation, daily math journals, a pre-test and post-test of math symbol recognition and math equation nomenclature and AIMS Web progress monitoring probes. The research showed that many students grew in their recognition of math symbols and equation nomenclature and also saw gains in their completion of math tasks on the progress monitoring probes. Research also found that students were reluctant to discuss their learning with one another and preferred conferences with their teacher. Further development will occur with modeling of math conversations.

Keywords: Math vocabulary, Montessori, math symbol recognition

Literature Review

Change the Equation, a non-profit, non-partisan coalition committed to improving science, technology, engineering, and mathematics (STEM) learning for every child, worked with Ogilvy PR to conduct a survey to get a feeling for Americans' attitudes toward math skills. According to the survey published on changetheequation.org (2014), one in five Americans report feeling frustrated or inadequate when they have to do math. According to the same survey group, nearly all (93%) agree that developing good math skills is essential to being successful in life. Where is the disconnect? With nearly five of five people agreeing that good math skills are essential to being successful in life and one in five feeling anxiety in doing math, how can we bridge that gap? This concern led me to focus my research on supporting the development of math vocabulary and operation recognition strategies in my E1 classroom through learning activities that I implemented in my classroom. These strategies will help students recognize and apply math operations to math they encounter in their lives. My hope was that in building that vocabulary and recognition, the anxiety and unknown regarding applying mathematics would diminish and students would more confidently and skillfully solve mathematical problems in their daily lives.

Strategies

Teaching vocabulary and symbol recognition.

Devoting time during different parts of the day towards introducing new vocabulary that will be used in a lesson or reviewing words that students already know is an excellent strategy towards building math vocabulary. Different times of the day that are recommended for vocabulary introduction are in a question of the day, morning message, homework prompts, read-alouds or during the lesson (Bay-Williams, 2009; Brett, 1996; May, 1995). When introducing new vocabulary, it was often mentioned that the exposure is not isolated, that the exposure should be repeated and the vocabulary should be

consistent (Bay-Williams, 2009; Brett, 1996; Gay, 2002; May, 1995). Weaving vocabulary into various parts of the day and teaching intentionally is an outstanding tool to help reinforce new concepts. In a Montessori classroom, story-telling is also a strong method to reinforce vocabulary. The story, The Four Strange Brothers by Michael Dohrer (2014) is just one example of a great mathematics story that can be told by a guide and retold by children to help reinforce math vocabulary.

Some different strategies for practicing math vocabulary are helpful for retaining knowledge and include sight recognition cards (May, 1995), verbalizing the new vocabulary and using in a sentence (May, 1995; Bay-Williams, 2009; Gay, 2002) and making visual word associations (Gay, 2002). A visual word association is a diagram where there are four rectangles attached to one another like this and words are entered:

Equation (the new vocabulary)	Can be written left to right or up and down (personal example of the word)
An equation is a number sentence with an equal sign (definition of the vocabulary)	2+4=6 (picture that describes vocabulary)

Figure 1. Visual word association

I predicted that by employing these strategies in the classroom, use and understanding of vocabulary to describe mathematical concepts would increase.

Putting the vocabulary into plain, simple language that students can relate to is a main focus of math vocabulary building and helps students to discover that math is part of their world. Recognizing vocabulary that may have multiple meanings and that are culturally relevant (Bay-Williams, 2009) is particularly essential for ELL students. It is essential not to overwhelm students with too much information at one time by planning instruction using the 5 C's—isolating **concepts**, identifying **content** or subject matter words, identify words to be **clarified**, identify words to avoid or **cut**, and identify words that should be **constructed** or taught (Smith, 2012).

According to John A. Van De Walle (2004), math symbols should be introduced in the context of a word problem. For instance, you could introduce Eric had 12 eggs and 4 broke, how many eggs were

left over? Here is a way we could write that 12-4=8. The minus sign should be called either minus or subtract but not as take away because we are not actually using manipulatives.

Also of note, special care should be taken with the "=" sign. "An equation such as 4+8=3+9 has no answer and is still true because both sides stand for the same quantity. A good idea is to often use the phrase "is the same as" in place of or in conjunction with "equals" as you read equations with students" (Van De Walle, 2004, p. 139).

Practicing math with daily activities.

Carolyn Moore (1995) discovered identifying vocabulary that needs to be retaught through interview is an invaluable tool to inform instruction. Moore also mentioned some daily activities that she has used in her classroom that I find intriguing and beneficial in practicing math including writing math stories and including math skills such as weighing, measuring, estimating, graphing, counting, comparing, and sorting along with the writing. She has also used daily calendar activities to emphasize place value, odds and evens, and monetary value as it relates to the date – for example, $4.04 for April 4. The class then talks about what coins they could use to make that dollar amount. When they make those observations in their math journal, Moore asks them the questions "How did you find that answer? What was your strategy? Can you show us that in a drawing? Can you explain your understanding in writing?" (Moore, 1995, p. 51).

Math journals.

Although it is time consuming, for both children and the guide, as written by Kathleen Kostos and Eui-kyung Shin "math journals can be a beneficial tool for my students and myself as a teacher" (Kostos & Shin, 2010, p. 230). Kostos and Shin (2010) emphasize the benefits of math journals noting

that “a review of students’ journals provided information about concepts that needed to be re-taught to individual students, groups of students or the whole class” (p. 229).

Kostos and Shin discussed how mathematical thinking can happen in many different modalities—written, oral, pictures and using manipulatives and through teacher questioning; using additional probing questions to encourage deeper thought and challenge students. Their mission was to have students “effectively communicate their mathematical thinking, so my students can explain how and why they use a mathematical concept to solve a problem” (Kostos & Shin, 2010, p. 224).

Using digital cameras to practice math concepts.

This project incorporated counting objects and matching the number of concrete objects with the printed numbers. The mission of the class was to work in groups and group objects and photograph them from 1 through 100. They would work on putting the objects into groups of 10 and then count the units that were left over.

Using storytelling and picture books to grasp meanings of concepts.

Lynn Columba, an associate professor in the College of Education at Lehigh University has studied and is interested in finding picture books that develop mathematics concepts and promote mathematical discussion by students. In her article related to introducing multiplication facts to third graders, she introduces sound strategies including having students find patterns, use manipulatives, and building on facts they already know (Columba, 2013). Columba provides a nice list of children’s literature that can build the knowledge of doubles facts, fives facts, zeros and ones facts, squares, sixes facts, and nines facts. (Appendix H)

When a student’s interest is sparked with a story and that student is able to use manipulatives, discuss their thinking strategies with classmates, it “offers a winning combination for mastering

multiplication facts based on conceptual ideas and relationships" (Columba, 2013, p. 374). Columba goes on to say that "only after students have an understanding of the meanings of the operations and symbols can the process of memorization begin" (Columba, 2013, p. 380).

One of the cornerstones of my introduction to the operations symbols in my classroom is to tell the story of the Four Strange Brothers. My mentor teacher, Markell Lockwood, shared this story with me after she attended a storytelling workshop (Dohrer, 2014). She typed up the story and shared it with me and I proceeded to tell the story to my entire class that week. The story talks about 4 brothers who wear different colors. The brother wearing red likes to count everything one by one. The brother wearing yellow groups things together and skip counts to save time. The brother wearing green likes to make the total amount smaller, so usually takes from the whole and puts a bit to the side. The brother who wears blue is the most loved of all—he likes to make sure everyone gets their fair share.

Building Fluency.

Linking math fluency to untimed math and reading performance in twins.

There is research that indicates significant genetic overlap among reading and math fluency, finding that genetics are a major factor on math performance (Petrill, Logan, Hart, Vincent, Thompson, Kovas, & Plomin, 2012). This study also found that timed math tests are valid and important gauges of learning, but also mentioned that untimed math tasks are also quite beneficial in "targeting students' strengths and weaknesses across these domains" (Petrill et al., 2012, p. 380).

Success of different strategies used to memorize math equations.

How do people who are successful at math calculate the answer when dealing with memorization facts? As defined in one research study, there are four different ways that students are

successful in answering math fact problems. The first is that they just have the fact memorized. The second is that they have a fact memorized and can easily apply what they know to another problem (example: I know 10+10 is 20 and that 9+8 is 17 because 9 is one less than 10 and 8 is two less than 10, when you add one plus two and take that away from 20, it is 17). The third is that a student counts on—6+7—where you start counting up from 7—8, 9, 10, 11, 12, 13—you counted 6 up from 7. The fourth strategy is called counting all where you start counting at the beginning for both numbers 6+7 1,2,3,4,5,6,7,8,9,10,11,12,13 (Gray & Tall, 1994). The research found that most children who are seven years old and are considered by their teachers to be strong math students use a strategy that is not just immediate recall of the fact. By the time a student is 12 years of age, all "strong" math students, as defined by their classroom teacher, are no longer using the count on or counting all strategies; they either have immediate recall or derive their knowledge from knowing other facts.

Building fluency, constructing knowledge and having fun.

Jo Boaler (2015), a professor of Mathematics Education and co-founder of YouCubed at Stanford University has written a helpful article speaking of increasing mathematical fluency through building number sense, having number talks, activities that build addition and multiplication fact knowledge and a really neat activity called number cards.

What is number sense? Number sense has been defined as how comfortable someone is in using numbers and their ability to apply to real world situations. If a student is simply memorizing facts and not really placing any real world context, they lose the impact of what the number really means. **What are number talks?** Number talks were developed by Ruth Parker and Kathy Richardson and are a "short teaching activity that teachers can start lessons with or parents can do at home. It involves posing

an abstract problem such as 18x5 and asking students to solve the problem mentally. The teacher then collects the different methods and looks at why they work" (Boaler, 2015). **What are math cards?** Math cards are representations of different ways you can arrive at a solution for a math sentence. An example for 9x4 you could #1 write an array of 9 columns by 4 rows of squares, #2 write 4x9, #3 have 4 pictures of dominoes that each have nine dots on them, or #4 have nine dice with the number 4 on each dice

Conditions for Learning.

Effects of Anxiety on Math Performance.

As I have been collecting data, I found a body of research conducted by Ramirez, Gunderson, Levine and Beilock (Ramierz, Gunderson, Levine, Beilock, 2013) who studied the correlation between math anxiety, short term memory and math achievement in the early elementary school.

These researchers developed an anxiety scale with three different facial expressions—a crying face, a medium face and a happy face. They paired the scale with a list of eight questions and administered it to first and second grade students from five public schools in a large urban school district. The study found students who had a higher working memory are more susceptible to math anxiety.

Self-Administration of math interventions.

Hulac, Dejong and Benson studied a group of 12 fourth graders in a small mid-western school setting with multiplication facts. Two graduate assistants with graduate-level training in academic interventions taught the students the intervention and oversaw the process. Once the students were able to demonstrate they were able to complete the 17 step intervention process successfully, the students were able to self-administer their intervention. The 17 step process involved the use of flash cards and developing a pile of unknown and known facts from the stack of flash cards. From these

stacks, 7 known facts and 3 unknown facts are put into a daily stack. The goal is to make the 3 unknown cards known and be able to move them to the known fact pile. The interventions were held twice per week for 20 minutes each session (Hulac, Dejong & Benson, 2012).

The results of the intervention were positive. 7 of the 11 students demonstrated faster growth in the intervention phase as compared to the baseline phase. This prompts me to try this intervention in my classroom and to employ mixed operations in the deck of flash cards to strengthen attention to operation symbols.

.

Concluding Remarks

In the review of literature, three main themes emerged. First is teaching vocabulary and symbol recognition. Some key strategies that I used for this theme were the use of math journals and using storytelling and picture books to grasp meanings of concepts. The second theme that emerged was the concept of building fluency. Some ideas that I kept in mind included the link between math fluency and reading performance, observing students using different strategies to solve math equations and also the concept of having fun while building fluency. The third theme identified is providing optimal conditions for learning; keeping the idea in mind that some children suffer from anxiety in their math and I should search for ways to help ease their anxiety. The second idea in the conditions of learning theme involved the use of self-administered math interventions. This is a strategy I plan on implementing in the future and will be discussed in further detail in my reflection.

Math has been identified by nearly all Americans as being an essential skill to be successful in life. One in five of those same Americans feel anxiety over performing math in their daily lives. The

anxiety may stem from tipping great service at a restaurant, it may be at a sale for 33% off a new television on Black Friday, or it may be the 20% increase in health insurance in the coming year.

Introduction

The purpose of this action research project is to develop tools and systems that build math vocabulary and recognition of the type of math operations required to solve designated problems in a Montessori E1 classroom. As a Montessori teacher in a lower elementary classroom, I strive to give children an introduction to math operations, vocabulary and symbols and to ignite their curiosity and enjoyment of solving mathematical equations. In the transition between the first and second planes of development, the child is moving from purely sensorial experiences with math materials toward abstract and imaginary thought. They are also seeking intellectual independence. When introducing concepts and observing children working with the materials, I am striving to guide them to be independent in their problem solving. In my research, I identified five essential questions related to my quest to build math vocabulary and math operations recognition. My key question was:

- What materials would be beneficial to help students recognize math operations symbols?

Additional questions included:

- How do I support the building of math vocabulary in my classroom?
- What types of math materials can I develop that feature mixed operations?
- What lessons and materials can be developed to help students apply math operations to real life situations?
- What methods can I introduce and model to help students explain their thought processes with their peers?

Through my action research, my goal was to gather tools and systems for Montessori guides to implement in their own classroom environment. These tools and systems will ideally invite students to recognize math operations, build math vocabulary, apply mathematical thinking to real life situations and enable them to explain their thinking to others. My goal is in line with Montessori's insistence that "our aim is not only to make the child understand, and still less to force him to memorize, but so to touch his imagination as to enthuse him to his innermost core"(Montessori, 1989, p. 6).

Methodology

Setting and Participants

This study took place at a public Montessori elementary school in a small upper Midwestern city. The school has a population of about 165 students, with a demographic of 93.83% Caucasian, 2.47% Hispanic, 2.47% African American and 1% Asian. Approximately 20% of the population qualifies for free and reduced lunch. The school consists of two Children's House classrooms (4K and Kindergarten), four Lower Elementary or E1 classrooms (a multi-aged classroom with grades 1-3) and three Upper Elementary or E2 classrooms (a multi-aged classroom with grades 4-6). There are some students with special needs who attend this school by parent choice. These students mainly receive support from the classroom teacher. For students with a formal IEP, in this case, speech and ELL, support staff travel to the school or the student may qualify to be served at a surrounding school.

Data was collected from students for a three month period of time starting in January 2015 and ending in March of the same year. At the beginning of the study, 19 students were a part of the

classroom and during the study, one student moved out of district leaving 18 students in the classroom; seven Alpha students or 1st graders (3 girls and 4 boys), seven Beta students or 2nd graders (2 girls and 5 boys), and four Gamma students or 3rd graders (2 girls and 2 boys).

Materials

The materials used in this research include addition equations and sums box, subtraction equations and differences box, multiplication equation and products box, division equation and dividends box, student math journals, math journal directions for daily activities related to numbers of the week for our beta and gamma students, daily math prompts featuring mixed operations for our alpha students, 3-part cards devoted to matching operation symbol with the word and description of the operation and 3-part math operation nomenclature cards.

The math equation boxes are pictured. Each operation has a control chart where a student can self-correct or work with a partner to check work.

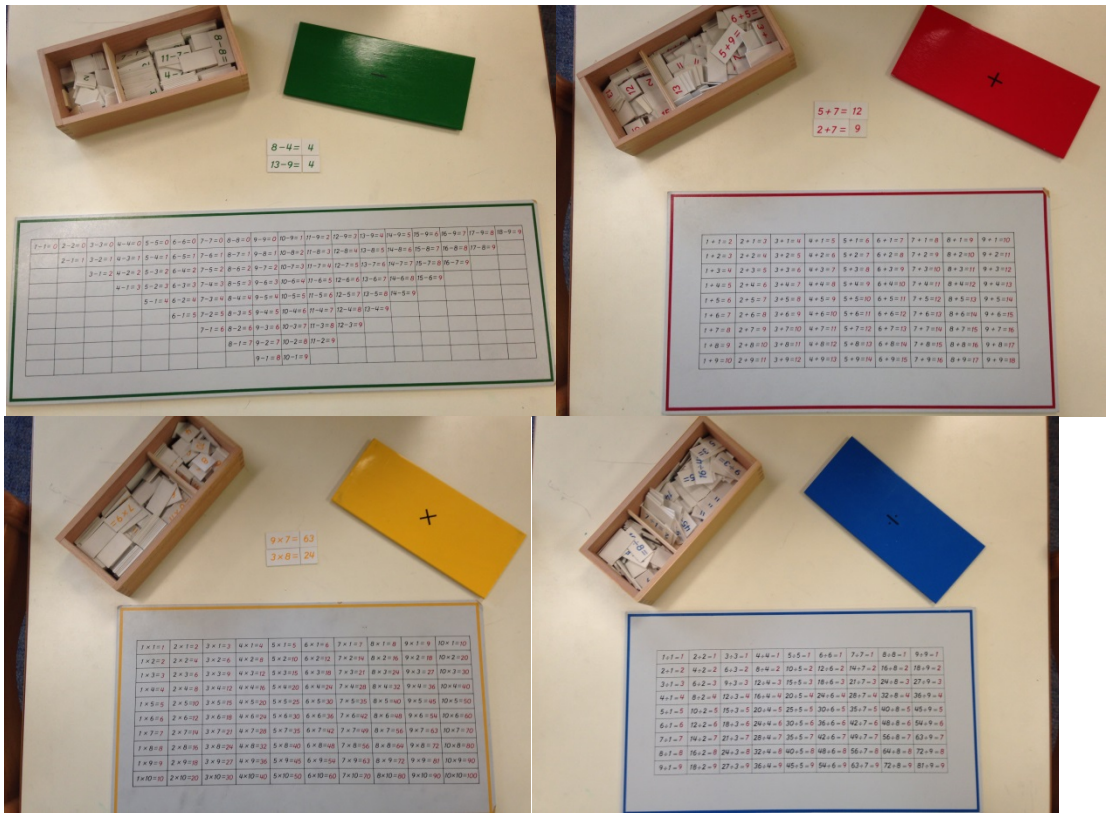
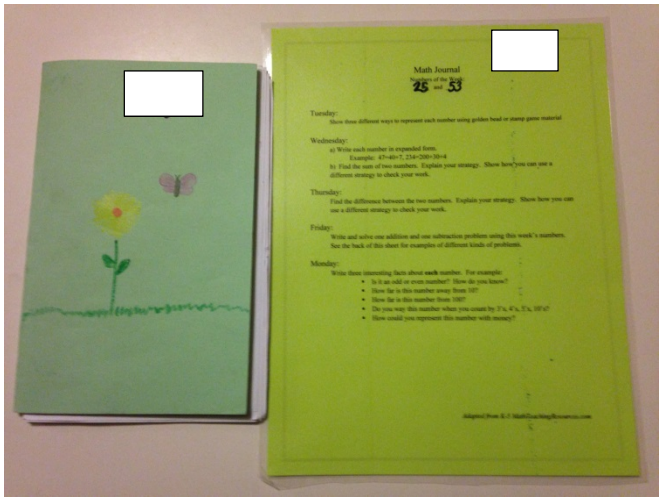


Figure 2. Math equation boxes and control charts

An example of a student math journal and math journal directions are pictured. The math journal directions feature two numbers of the week and have different tasks assigned with those two numbers each day. The math journal directions were adapted from K-5 Teaching Resources website www.k-5mathteachingresources.com/ (Appendix G).



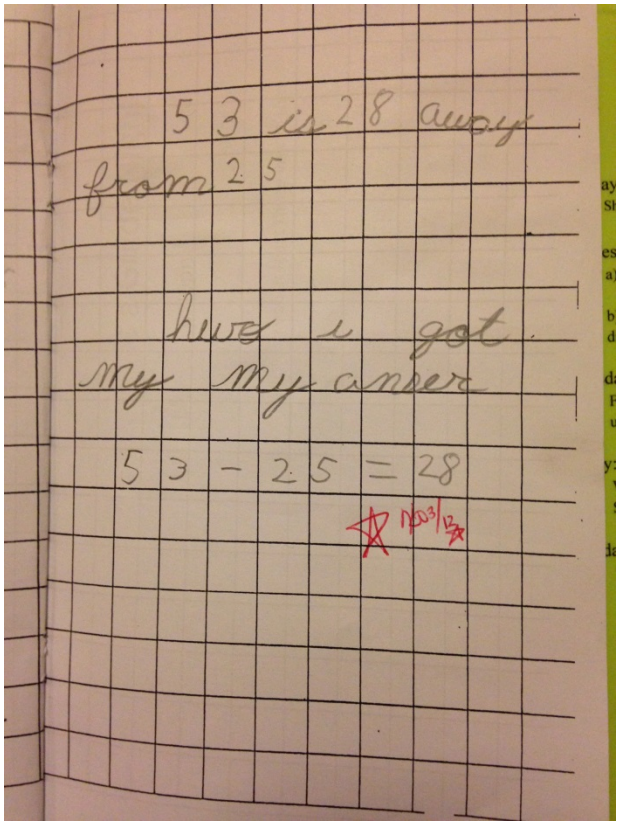


Figure 3. Math journal

Daily math prompts were copied from the white board into math journals and solved each morning by our alpha class members. Here is a sample of their work:

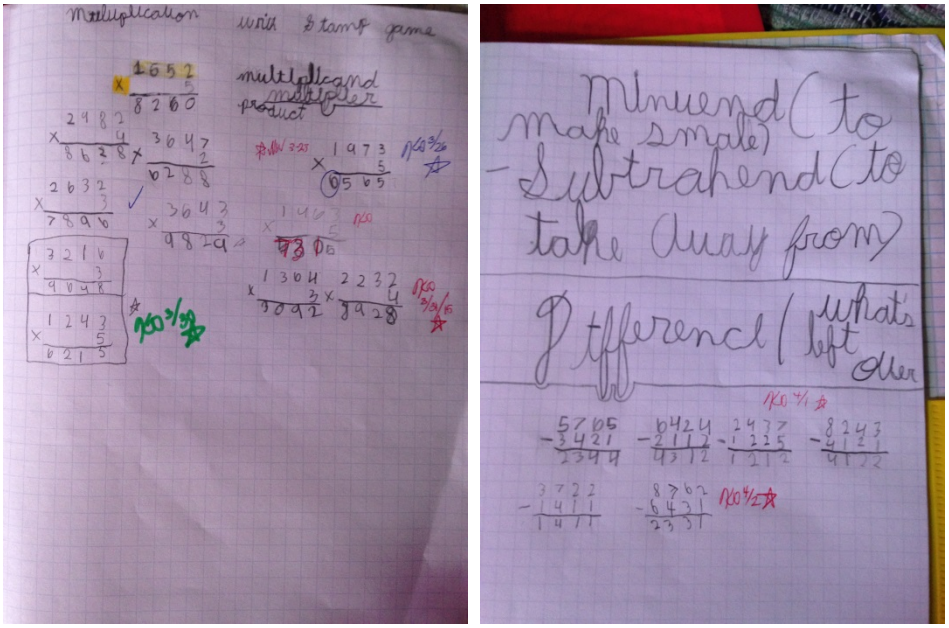


Figure 4. Daily math prompt

The final materials used were 3-part cards dedicated to matching operation symbols with description of the operation (Appendix A) and the 3-part nomenclature cards (Appendix B). Students were introduced to the symbols and nomenclature of the operations along with hearing the story of The Four Strange Brothers (Appendix C) colors that correspond with the self-correcting nature of the material.

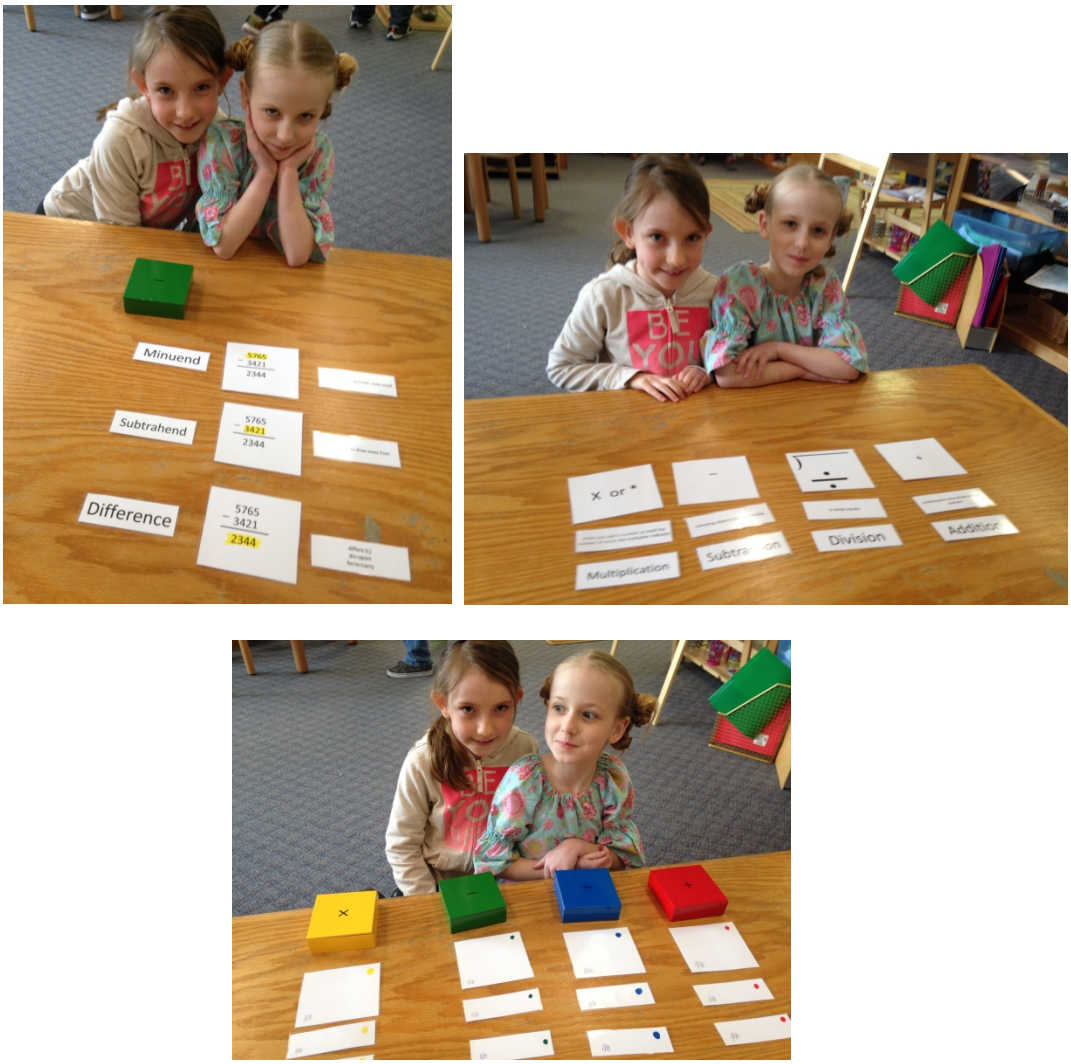


Figure 5. Children studying vocabulary

Procedure

Research was conducted by collecting data on a pre-test and post-test on math operations vocabulary, daily math journals featuring numbers of the week for Beta and Gamma students, daily math prompts featuring mixed operations for Alpha students, and weekly progress monitoring

AIMS/WEB progress monitoring probes for all students. Additionally, observations were made, anecdotal records were collected and case studies were recorded.

In this lower elementary Montessori classroom, new math lessons were given once per week to grade level groups. To practice their math during the week, students use Montessori materials daily to find answers to their assigned work. In addition to regular lessons, students are responsible for completing daily math tasks. The Alpha group (ages 6-7) was given a daily math prompt that was written on the white board each morning, discussed at morning meeting and then solved as one of their first works of the day. The Beta and Gamma groups (ages 7-9) were assigned a math journal page each week on Tuesday. The reason Tuesday was considered the first day of the week is because math lessons are taught on Tuesday. The students were assigned the same format of daily math for 12 weeks. As students completed their work, they would check in with me and describe their thinking.

Students' completion of task rates for daily math journals and daily math prompts were tracked over the period of 1/5/15 to 3/31/15. This data along with observational data enabled me to observe patterns and connections between observed student engagement with dialog/collaboration with other students and completion of tasks. A grade-level class average was found for all three grade levels in order to determine whether or not increased dialog/collaboration led to higher task completion rates.

In addition to the tracking of daily math journals, each Tuesday, I administered a Progress Monitoring probe from AIMS/WEB to the entire class (Appendix D, E and F). These probes feature mixed operations and are written for specific grade levels. The test is administered at the same time each week and is administered in place of our typical math fact testing called *Rocket Math*, which features one operation at a time. Students test scores for the AIMS/WEB progress monitoring was tracked over the period of 1/12/15 to 4/7/15.

Nomenclature 3 part cards were introduced in early January at each lesson (Appendix A). Students were introduced at all grade levels at each lesson to the etymology of the math operations. Words such as addend, sum, minuend, subtrahend, difference, multiplicand, multiplier, product, dividend, divisor and quotient were defined and labeled as lessons were presented. In addition to the nomenclature cards, 3 part cards were also introduced with operations symbols, name of operation and a brief description of the operation (Appendix B). For all three part cards, a control of error was included and color coded in accordance to the color used on the math equation boxes for specific operations and the colors from the *Story of the Four Strange Brothers (*Dohrer, 2014). *The Story of the Four Strange Brothers* (Appendix C) is a delightful story that I introduce shortly after telling the story of numbers. After telling this story, the children enjoyed retelling the story and also noticed that the memorization material is colored in the same pattern—red for addition, yellow for multiplication, green for subtraction and blue for division.

A pre-test of knowing etymology of math operations and symbol recognition to the name of the operation were conducted in January 2015. A post-test was administered on April 7, 2015. Results of the two tests were compared to gauge the effectiveness of the two strategies to build math vocabulary and operations symbol recognition.

Lastly, for the final 15 minutes of instructional time on Tuesdays and Thursdays, 15 students engaged in a math activity called math fact practice where students will choose either a box of math equation boxes or a ring of math fact cards. They quiz one another on math facts and use the checking boards to make sure their partner is on the right track. Students do correct one another if they find their partner is not secure in their math facts. No data collection is done at this point, just observations were made.

Three students were involved in math interventions with one of our paraprofessionals twice per week for 15 minute periods. They worked on addition math facts starting in November. Two of the

children are third graders and one of the children is a second grader. The intervention was designed to have the three students work with a deck of addition flash cards to find addition facts they know and addition facts that are unknown.

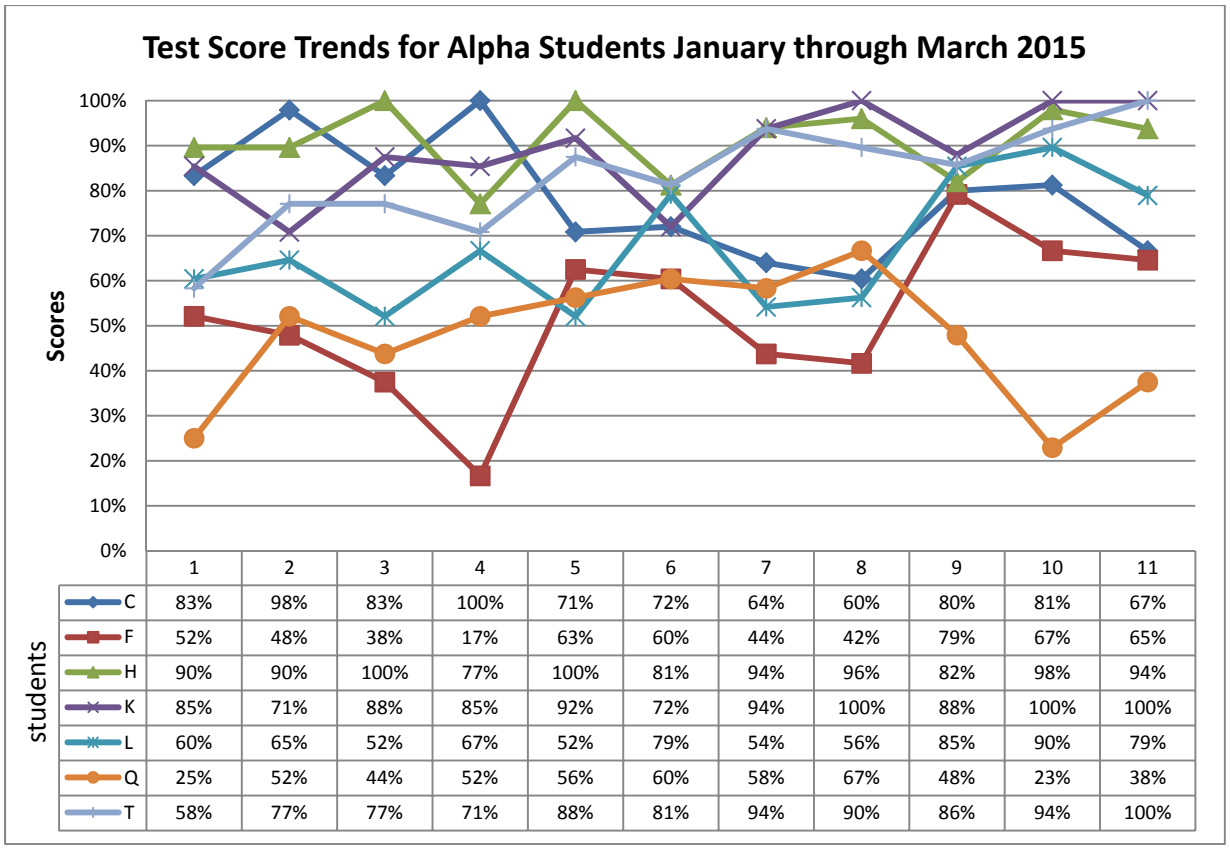
To determine whether or not the facts are known/unknown, the paraprofessional times the student and if they can recite the correct answer in three seconds or less, the card is put into the known pile. If they take more than three seconds to answer or are incorrect, the fact goes into the unknown pile. My original goal was to have the two third grade students master addition facts and be able to move on to multiplication facts and for the second grader to move from addition to subtraction.

Results

What is the effect of the daily math journals, mixed daily math operations, three part cards and math fact practice? According to the AIMS Web progress monitoring, the results are positive for some students and unclear for others (Figures 6, 8 and 10). While some students grew in their recognition of symbols in their weekly progress monitoring, others performed at levels that were inconsistent.

For students and their recognition of symbols and etymology, the results were more clear and promising with a majority of students growing in their recognition of the mathematical terms related to the three math operations of addition, subtraction, and multiplication (Figures 7, 9 and 11). When I introduced the 3 part cards, at least half of the students were able to retell the story of the Four Strange Brothers. The impact of storytelling the classroom is incredible.

In tracking students and their completion of tasks for both daily math journals and mixed operation daily math prompts, it was found that children completed the tasks 97 percent of the time. Their understanding and ability to articulate how they arrived at their understanding is a work in progress and something that I plan on further investigating and modeling in the future of our classroom.



students	1	2	3	4	5	6	7	8	9	10	11
C	83%	98%	83%	100%	71%	72%	64%	60%	80%	81%	67%
F	52%	48%	38%	17%	63%	60%	44%	42%	79%	67%	65%
H	90%	90%	100%	77%	100%	81%	94%	96%	82%	98%	94%
K	85%	71%	88%	85%	92%	72%	94%	100%	88%	100%	100%
L	60%	65%	52%	67%	52%	79%	54%	56%	85%	90%	79%
Q	25%	52%	44%	52%	56%	60%	58%	67%	48%	23%	38%
T	58%	77%	77%	71%	88%	81%	94%	90%	86%	94%	100%

Figure 6. Progress monitoring trends for alpha students January through March 2015

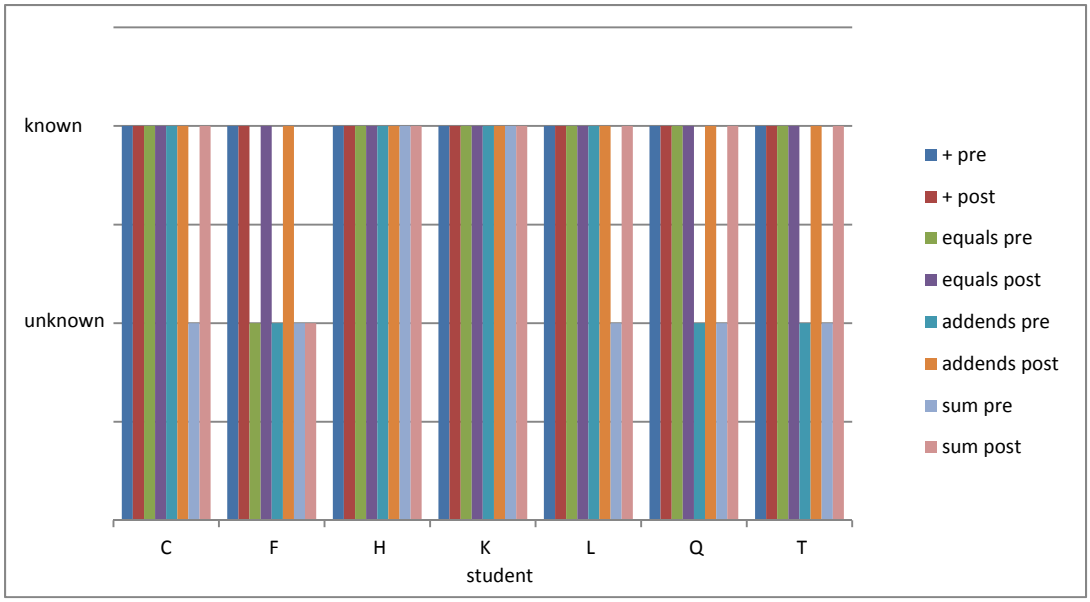


Figure 7. Alpha group pre and post math vocabulary and symbol recognition results for addition

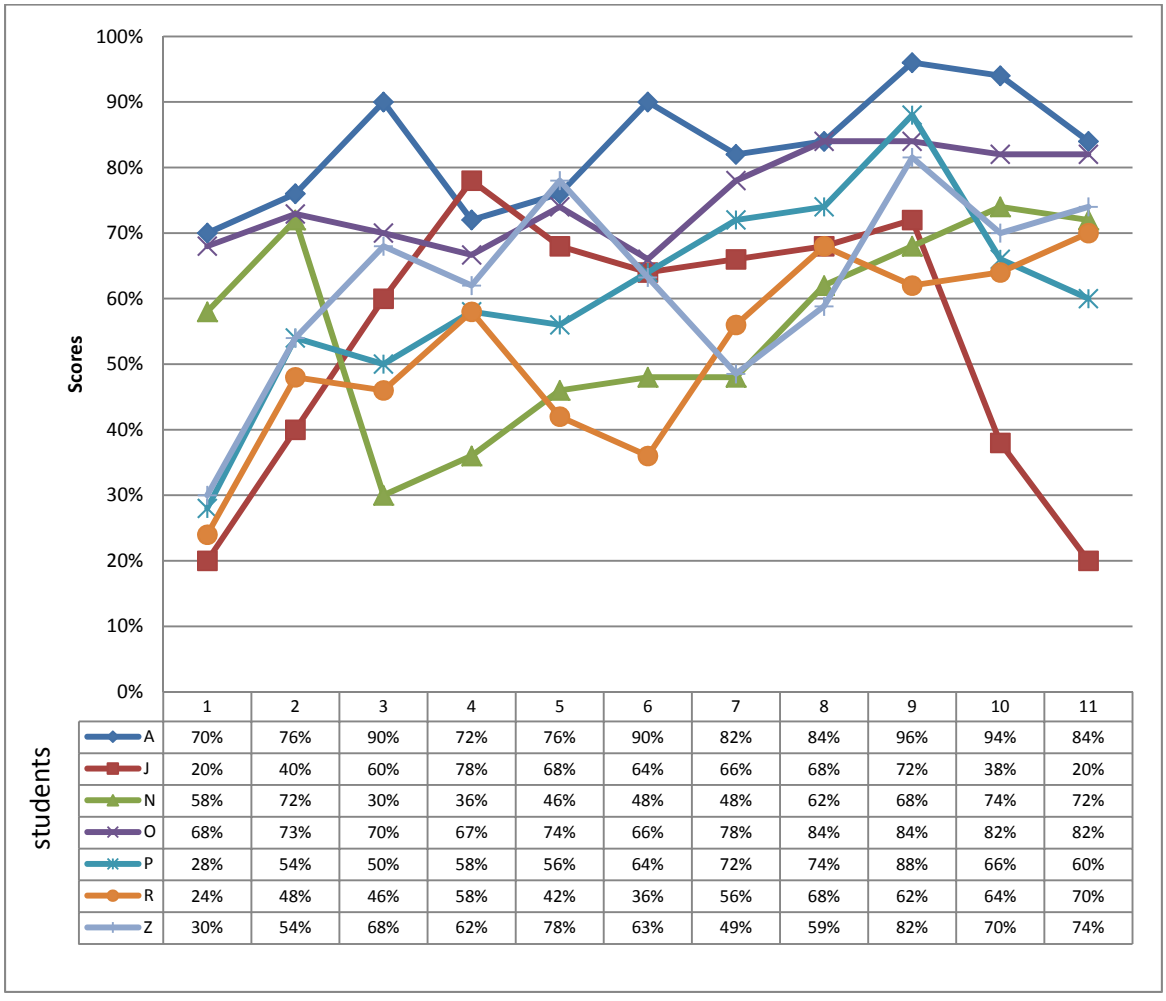
This particular group of Alpha students is really strong in math. With my action research project and with my natural interest in mathematics, I have been working a lot with math this year, but this group is going to go far in their Montessori math experience.

Looking at AIMS/WEB progress monitoring results (Figure 6), 5 of 7 Alpha students showed gains in their scores of 10 percent or more ranging from 13-42% increase showing a strong gain in symbol recognition and application of the correct operation. When exploring the pre and post test results of the Alpha students and their recognition of "x", "=" signs and the vocabulary of addends and sum (Figure 7), two of the seven students were able to identify all components in January and in April. 6 of 7 students were able to identify or explain the symbols or vocabulary on the last day of the study. The 7th student was able to identify or explain 3 of the 4 elements of an addition equation, but could not identify the sum.

`There is one particular Alpha student "Quincy" or student Q (see individual results in figures 6 & 7) in the Alpha class who is very inconsistent in his ability to demonstrate his math knowledge. Most days, Quincy is capable of completing his daily math and math work from the weekly lesson, but there are days when he comes to me and asks me to show him how to set up an addition problem with the golden beads or what material he can use to answer a problem like 12+4. In our twice weekly math fact practice sessions, I have observed Quincy applying the incorrect operation to a subtraction or addition fact. His inconsistency is evidenced by his performance on the AIMS Web progress monitoring. Quincy did grow in his math language quite nicely, so I'm hoping his foundation in knowing what the numbers are in an operation and recognition of symbols will help him in his future.

Another first grader that I have chosen to further study is "Charlie" or student C (see individual results Figure 6 & 7) from the Alpha class. Charlie is a strong performer and has a competitive streak. He often completes his daily math so he can be the first to have it checked. In addition to the interventions

mentioned in my action research, our class engages in a daily math fact testing called Rocket Math. Charlie has expressed concern to both his parents and me if he doesn't pass a Rocket Math in one try. He has told me "my goal is to finish multiplication this year." Charlie was on track to get to multiplication, but there is a hurdle of some more challenging subtraction work that he needs to finish first. My observations are that Charlie is getting frustrated with not achieving his goal in Rocket Math so is very quickly answering the progress monitoring probes (he has eight minutes to complete and he's turning in after about four minutes) and is making computational errors. He is recognizing symbols, but he is not taking his time and re-checking his work. That is something that I will be discussing with him in the coming week.



students	1	2	3	4	5	6	7	8	9	10	11
A	70%	76%	90%	72%	76%	90%	82%	84%	96%	94%	84%
J	20%	40%	60%	78%	68%	64%	66%	68%	72%	38%	20%
N	58%	72%	30%	36%	46%	48%	48%	62%	68%	74%	72%
O	68%	73%	70%	67%	74%	66%	78%	84%	84%	82%	82%
P	28%	54%	50%	58%	56%	64%	72%	74%	88%	66%	60%
R	24%	48%	46%	58%	42%	36%	56%	68%	62%	64%	70%
Z	30%	54%	68%	62%	78%	63%	49%	59%	82%	70%	74%

Figure 8. Progress monitoring trends for Beta students January through March 2015

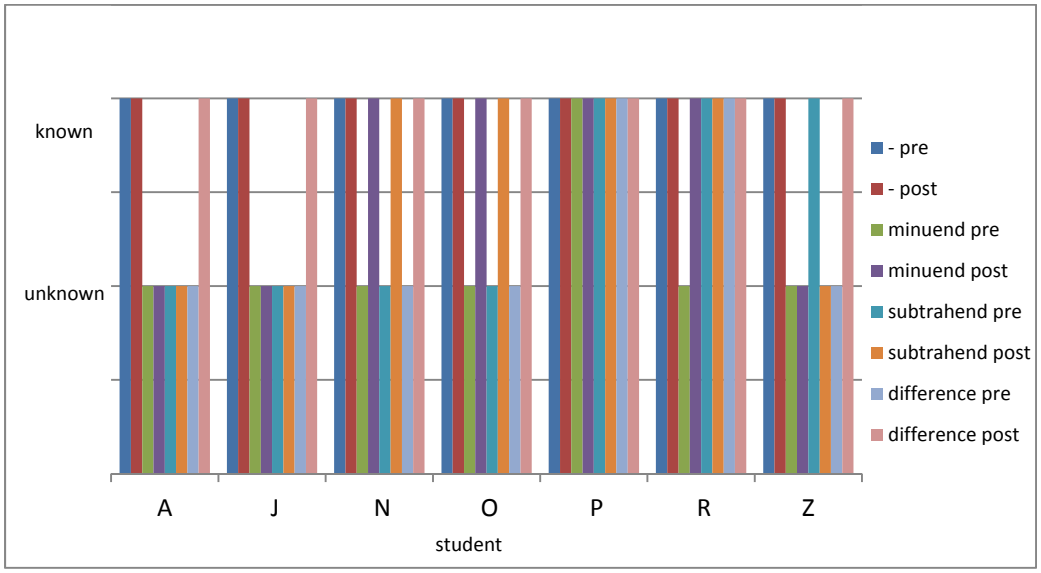


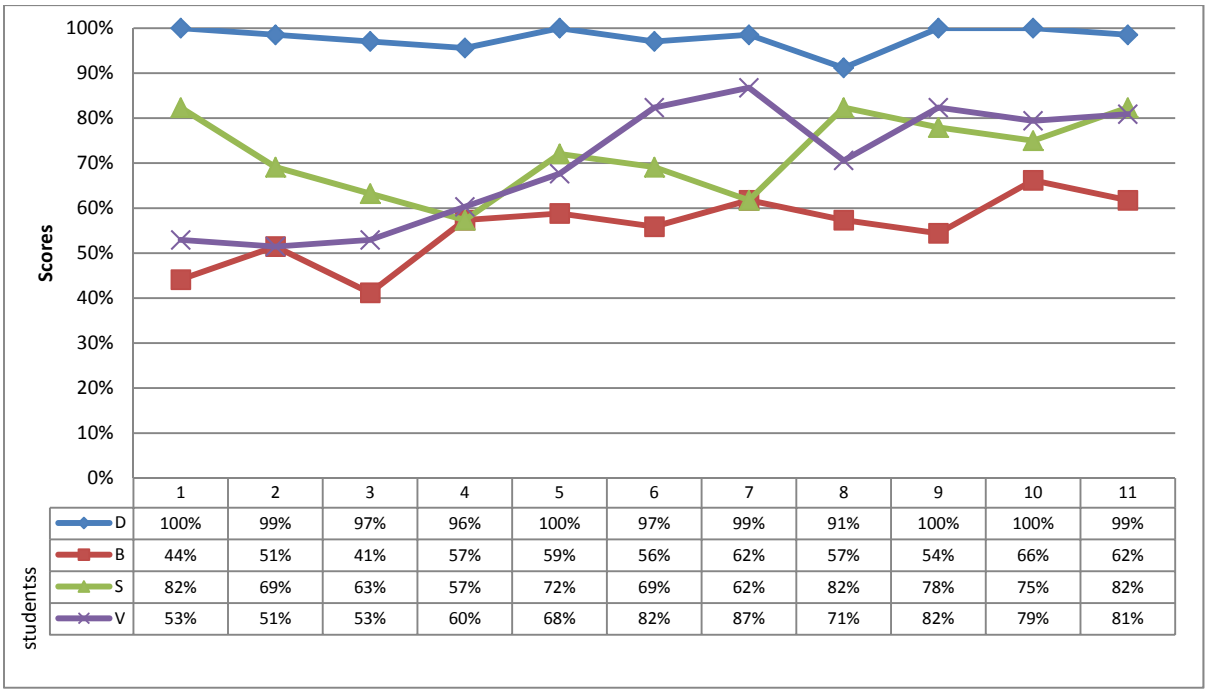
Figure 9. Beta group pre and post math vocabulary and symbol recognition results for subtraction

Exploring the AIMS/WEB progress monitoring for the Beta class (Figure 8) exposed gains of 10% or more in operation recognition for 6 of the 7 students. The range of gains for these six students ranged from 14 to 46 percent. Again, this is showing a nice improvement in symbol recognition and application of the recognition. When reviewing the results of the pre-test and post-test results for math vocabulary and symbol recognition (Figure 9), all 7 students were able to explain the operation of subtraction at the beginning and end of the study. The vocabulary of minuend and subtrahend were a bit of a sticking point with two of the seven students, they switched the names of the two terms. One student was secure in their discussion of subtraction, and the vocabulary of minuend, subtrahend and difference throughout the study.

A student who was inconsistent in his performance in progress monitoring is student J (see individual results on Figures 8 & 9) from the Beta class named “Jeremy”. Jeremy is a student who struggles with daily work completion and has been experiencing some changes to his required daily work due to challenges with writing and expressing himself in written and peer conversation. He was growing until a point in mid-March and completing his work, writing in his daily math journal when

suddenly in late March, he started to slide back to his original score of 20%. His slide in scores is attributed to not finishing his test. For the questions he completed, he answered them correctly and typically applied the correct operation to any math attempted throughout the study. Jeremy is currently working on daily math word problems that are prepared and he just has to write the answer and show his work. Of 12 entries, he applied the correct operation to the word problem 11 times.

Another student who has been on my watch list this year is “Norah” student N (see individual results on Figures 8 & 9) from the Beta class. Norah is a twin with her sister being in a partner class in our school. Norah started the school year demonstrating strong understanding of math operations and symbol recognition. Historically, Norah has not been as academically strong as her twin, but she has performed well in relation to her grade level. Mid-year, I noticed Norah was not performing as well in either reading or math. She often crossed out problems and did not attempt to solve them in her weekly progress monitoring probe. Norah completed all of her math journal activities and would often engage in dialog that demonstrated understanding of concepts such as odd and even, representing numbers in money, finding the difference between two numbers and finding the sum of two numbers. After some conversations about trying things even if they seem scary and talking about problem solving, Norah has started attempting all problems on her progress monitoring probes. Her errors typically are related to computational errors, but she has grown from 58 percent to 72 percent in her progress monitoring probe score.



	1	2	3	4	5	6	7	8	9	10	11
D	100%	99%	97%	96%	100%	97%	99%	91%	100%	100%	99%
B	44%	51%	41%	57%	59%	56%	62%	57%	54%	66%	62%
S	82%	69%	63%	57%	72%	69%	62%	82%	78%	75%	82%
V	53%	51%	53%	60%	68%	82%	87%	71%	82%	79%	81%

Figure 10. Progress Monitoring for Gamma students January through March 2015

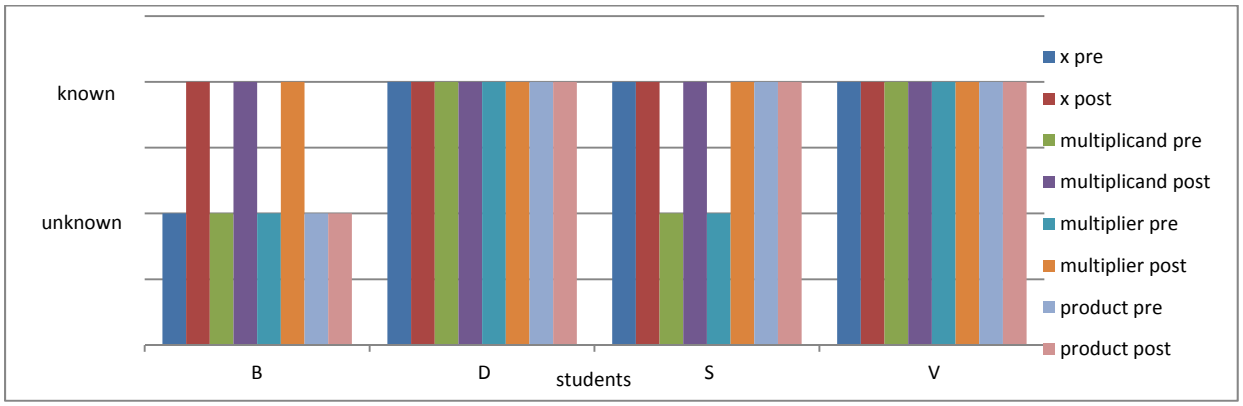


Figure 11. Gamma group pre and post math vocabulary and symbol recognition results for multiplication

Overall, the Gamma students are ending the year with a stronger grasp of multiplication and symbol recognition and operation application. According to the trends on the AIMS/WEB progress monitoring (Figure 10), two students improved their overall recognition of math symbols by 20% of the course of the study. One of the four students was able perform at or near a rate of 100 percent during the entire study. One student was inconsistent in his performance, but started and ended the study at the same point. According to the pre-test and post-test results of recognition of the “x” sign, and

multiplication vocabulary of multiplicand, multiplier and product (Figure 11), three of four students were secure in all aspects and the fourth was secure in all aspects except for product. The student who showed the most gains is the student who started the study not being able to explain or identify any components of multiplication.

The student who I first found to struggle the most and a large reason I pursued this study grew so much over the school year. “Brynn” student B from the Gamma class (see results on Figure 10 & 11) is a student I’ve worked with since she started second grade. She works really hard and has grown so much in both reading and math in the past two years. Brynn completed all of her math journal entries on time and wrote very clearly, expressing herself in both written and oral form. At times, Brynn would get confused by the terminology “difference” in her Thursday math task, but she would return to her work and come back with the realization of what difference meant. Brynn has also been working with our paraprofessional on mastering her math facts. She has mastered addition math facts and is starting to work on multiplication. Most of the errors she now is experiencing in her progress monitoring probes involve the operation of division. Once she masters her multiplication facts and sees the connection between the operations, I am confident she will find success.

In general, I am pleased with the results. It was not a perfect action research project, but I have learned a lot from the experience and look forward to explaining the next step in my reflection.

Reflections

The purpose of this action research project was to develop tools and systems that build math vocabulary and recognition of the type of math operations required to solve designated problems in a Montessori E1 classroom. There were several tools that I used to help build math vocabulary and math operation recognition. The **three part cards** introducing meanings of the different numbers and symbols in the four major operations were helpful when teaching weekly math lessons and really cemented my

own knowledge of “teacher talk” when working with learners. The **math journals** were excellent assessment tools to see if students understood those key vocabulary terms of difference and sum. They also gave students the opportunity to activate their prior knowledge of golden bead and stamp game values, odd and even knowledge, ability to write word problems, comparing two numbers with one another and skip counting, finding factors, translating numbers into coins and writing numbers in expanded form. **Daily math with mixed operations** for the first graders provided to be great practice for the students to switch gears and apply their knowledge of operations symbols appropriately. Lastly, the **math fact practice** with students practicing math facts with a partner at the end of the school day was so interesting to observe. Some students really enjoyed the challenge of pushing themselves to master new operations. I also observed some students who just liked to take it easy during this time and listen to their friends practice their math facts. It was great to see a collaborative and engaged group of students working together.

One component of this research that I would really like to explore further is the development of greater discussion between the students. Many students engaged me in conversation, but I rarely observed student engaging in math conversation with one another. The main observation I made is students asking one another for the answer and more often than not, the classmate was happy to share the answer with their classmate. Modeling math conversations will be a focus going forward.

Another area that I plan on pursuing is the math intervention mentioned in my literature review where there is a student led math intervention. The intervention seems like it would be in line with Montessori principles with being self-directed, thus fostering independence and providing a control of error where students are able to check their own work. Our school already employs a similar program with a paraprofessional administering fact practice twice a week with three students. If we could have

the entire class engage in the same intervention being self-directed, it would be interesting to see the outcome.

Lastly, I plan on educating families on the work that has been done in our classroom so far. I didn't know there were names like minuend and subtrahend before I started teaching in a Montessori classroom. As an adult, if I saw the word difference in this sentence "Find the difference between the two numbers", I don't know that I would have known that I would subtract the smaller number from the larger number. When there is consistent communication being heard by a student in the classroom and at home, it benefits all in the long run.

Appendix A

Quotient	Divisor	Dividend
$4\overline{)124}$ → 31	$4\overline{)124}$ → 31	$4\overline{)124}$ → 31
Quotien=how many times	Dividere= to force asunder	to be divided

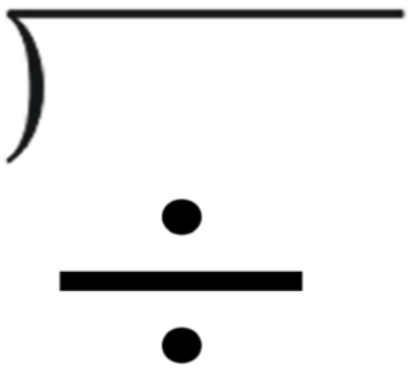
Multiplicand	Multiplier	Product
1652 × 5 = 8260	1652 × 5 = 8260	1652 × 5 = 8260
the number that is added to itself in multiplication	the number of times a number is added to itself	the result of multiplication

Appendix A page 2

Addend	Sum
4765 + 4823 9588	4765 + 4823 9588
addenda (L) = things to be added	summus (L) = highest, total

Minuend	Subtrahend	Difference
5765 − 3421 2344	5765 − 3421 2344	5765 − 3421 2344
minuere (L) = to diminish, make small	subtrahere = to draw away from	differe (L) dis=apart ferre=carry

Appendix B

Division	Multiplication
	X or *
to divide equally	when you add a number to itself the number of times the multiplier indicates

Addition	Subtraction
+	–
combining the value of two or more numbers	removing objects from a collection

Appendix C

The Story of the Four Strange Brothers

==========

Once upon a time there were 4 very strange brothers…

The eldest dressed entirely in red (describe).

On his cape he wore this emblem. (Show plus sign).

I'll take…..these two amounts and put them together to make a bigger amount!!!

I'll take these 3 amounts and put them together to make an even bigger amount!!!!

He did that when he visited friends, he did that at the grocery store, he did that at the bowling alley, he did that well, all over the place!

Isn't that strange…..

Well his brother dressed entirely in yellow…he liked to do what his brother did, but he would only add numbers of the same amount #. The brother would say, I'll take 5 and 5 and 5 and 5, 5 four times equals 20! I'll take 3 and 3 and 3 and 3 and 3, 3 five times equals 15!

He liked what his older brother did, but he wanted to be faster! His favorite shirt had this on it (show the multiplication sign).

Another brother dressed in green…he would always where a special pin that had this on it (show the symbol for subtraction).

This brother would go places and take things! He would say, I noticed you have 5 cookies sitting around, I'm going to take 3 away and leave you with only 2! That ____always has extra pencils she had 4 to spare so I took 2 and left her with…..

Strange I tell you!

The youngest brother was the town favorite. He dressed all in blue. He wore a tie that had this symbol on it. He liked to share things equally.

Well one day, there was a CATASTROPHE at the apple farm down the road. The 4 brothers came onto the scene……

Everything was a jumbled up mess! There were apples everywhere! The eldest brother came and calculated all of the piles of apples. He announced you have a total of 1,500 apples. He called that process: ADDITION.

Then the next brother came wearing the brightest of yellows. Remember he likes to do what his eldest brother does only faster, so he could see the groups of apples that were the same amount, so he used that number of apples and noted how many groups there were. He said 100 apples in each group and there were 15 groups. 100, 15 times = 1500. Let's call that MULTIPLICATION!

Then the 3rd brother came. He said I'll take this pile of 100 and leave a lot less!!!! We'll call that taking away, SUBTRACTION!

By this time there was quite a crowd of villagers gathered around to watch this spectacle!

Finally, the 4th brother showed up. He noticed all of the villagers standing there. Now, it's my turn he said.

Do you remember what he liked to do?

He handed out the rest of the apples equally to the villagers, one for you, one for you, etc. until all of the apples were gone!

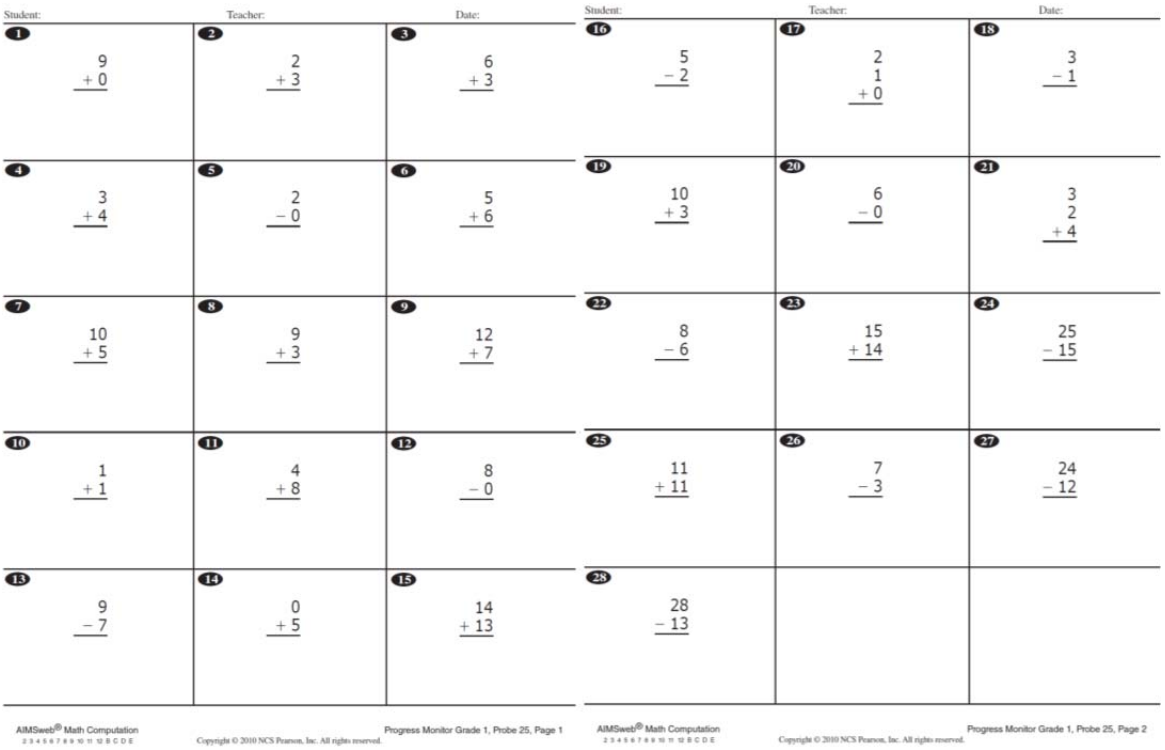
He said, now we call that DIVISION.

The town was left feeling very happy!!

We will be studying the rest of these operations throughout the year.

Story originally told by Michael Dohrer at University of Wisconsin River Falls Montessori Storytelling workshop. Retelling adapted and written by Markell Lockwood.

Appendix D



Student: Teacher: Date:

1) 9 + 0	2) 2 + 3	3) 6 + 3
4) 3 + 4	5) 2 − 0	6) 5 + 6
7) 10 + 5	8) 9 + 3	9) 12 + 7
10) 1 + 1	11) 4 + 8	12) 8 − 0
13) 9 − 7	14) 0 + 5	15) 14 + 13

AIMSweb® Math Computation
2 3 4 5 6 7 8 9 10 11 12 B C D E

Progress Monitor Grade 1, Probe 25, Page 1

Student: Teacher: Date:

16) 5 − 2	17) 2 1 + 0	18) 3 − 1
19) 10 + 3	20) 6 − 0	21) 3 2 + 4
22) 8 − 6	23) 15 + 14	24) 25 − 15
25) 11 + 11	26) 7 − 3	27) 24 − 12
28) 28 − 13		

AIMSweb® Math Computation
2 3 4 5 6 7 8 9 10 11 12 B C D E

Progress Monitor Grade 1, Probe 25, Page 2

Student: Teacher: Date:

Reminder: There is **no** partial credit when scoring. The answer must be correct **in its entirety** to obtain the correct score value. Note that the answer key provides the correct answers for each problem, but is not exhaustive. For more information and examples of alternative correct answers, please refer to the Scoring section of the M-COMP manual.

Grade 1, Probe 25 Answer Key

Item No.	Answer	Correct	Incorrect
1.	9	1	0
2.	5	1	0
3.	9	1	0
4.	7	2	0
5.	2	1	0
6.	11	1	0
7.	15	1	0
8.	12	1	0
9.	19	1	0
10.	2	1	0
11.	12	1	0
12.	8	1	0
13.	2	2	0
14.	5	1	0
15.	27	2	0
16.	3	2	0
17.	3	2	0
18.	2	2	0
19.	13	2	0
20.	6	1	0
21.	9	2	0
22.	2	2	0
23.	29	3	0
24.	10	3	0
25.	22	3	0
26.	4	2	0
27.	12	3	0
28.	15	3	0

TOTAL ☐

AIMSweb® Math Computation
2 3 4 5 6 7 8 9 10 11 12 B C D E

Progress Monitor Answer Key, Grade 1, Probe 25

Appendix E

Student: Teacher: Date:

1 $\begin{array}{r} 6 \\ +\ 8 \\ \hline \end{array}$	2 $7 + 2 =$ _____	3 $\begin{array}{r} 2 \\ +\ 1 \\ \hline \end{array}$
4 $\begin{array}{r} 14 \\ +\ 15 \\ \hline \end{array}$	5 $\begin{array}{r} 10 \\ +\ 7 \\ \hline \end{array}$	6 $4 - 3 =$ _____
7 $9 + 2 =$ _____	8 $\begin{array}{r} 11 \\ +\ 3 \\ \hline \end{array}$	9 $\begin{array}{r} 3 \\ 6 \\ +\ 8 \\ \hline \end{array}$
10 $\begin{array}{r} 16 \\ +\ 13 \\ \hline \end{array}$	11 $\begin{array}{r} 3 \\ -\ 2 \\ \hline \end{array}$	12 $\begin{array}{r} 73 \\ +\ 65 \\ \hline \end{array}$
13 $\begin{array}{r} 22 \\ -\ 11 \\ \hline \end{array}$	14 $\begin{array}{r} 30 \\ +\ 25 \\ \hline \end{array}$	15 $\begin{array}{r} 14 \\ +\ 3 \\ \hline \end{array}$

Student: Teacher: Date:

16 $\begin{array}{r} 3 \\ 2 \\ +\ 8 \\ \hline \end{array}$	17 $\begin{array}{r} 14 \\ -\ 2 \\ \hline \end{array}$	18 $\begin{array}{r} 8 \\ 3 \\ +\ 1 \\ \hline \end{array}$
19 $\begin{array}{r} 26 \\ -\ 25 \\ \hline \end{array}$	20 $\begin{array}{r} 36 \\ +\ 25 \\ \hline \end{array}$	21 $\begin{array}{r} 76 \\ -\ 70 \\ \hline \end{array}$
22 $\begin{array}{r} 39 \\ -\ 19 \\ \hline \end{array}$	23 $\begin{array}{r} 27 \\ -\ 23 \\ \hline \end{array}$	24 $\begin{array}{r} 2 \\ 3 \\ +\ 7 \\ \hline \end{array}$
25 $\begin{array}{r} 14 \\ -\ 7 \\ \hline \end{array}$	26 $\begin{array}{r} 30 \\ -\ 28 \\ \hline \end{array}$	27 $\begin{array}{r} 95 \\ +\ 99 \\ \hline \end{array}$
28 $\begin{array}{r} 81 \\ -\ 79 \\ \hline \end{array}$		

Student: Teacher: Date:

Reminder: There is **no** partial credit when scoring. The answer must be correct **in its entirety** to obtain the correct score value. Note that the answer key provides the correct answers for each problem, but is not exhaustive. For more information and examples of alternative correct answers, please refer to the Scoring section of the M–COMP manual.

Grade 2, Probe 25 Answer Key

Item No.	Answer	Correct	Incorrect
1.	14	1	0
2.	9	1	0
3.	3	1	0
4.	29	1	0
5.	17	1	0
6.	1	1	0
7.	11	1	0
8.	14	1	0
9.	17	1	0
10.	29	1	0
11.	1	2	0
12.	138	2	0
13.	11	3	0
14.	55	1	0
15.	17	1	0
16.	13	2	0
17.	12	2	0
18.	12	2	0
19.	1	2	0
20.	61	3	0
21.	6	2	0
22.	20	2	0
23.	4	2	0
24.	12	2	0
25.	7	3	0
26.	2	3	0
27.	194	3	0
28.	2	3	0

TOTAL ☐

Appendix F

Student: Teacher: Date:

1) 3 + 3	2) 11 + 4	3) 7 − 6	4) 11 + 7 = ____
5) 9 × 1	6) 19 − 6 = ____	7) 2 × 5	8) 10 + 14
9) 17 + 19	10) 4 × 3	11) 11 − 8	12) 6 × 8
13) 15 + 16	14) 7 × 9	15) 99 − 91	16) 55 + 39
17) 18 − 14	18) $4\overline{)8}$	19) $5\overline{)30}$	20) 17 − 13

Student: Teacher: Date:

21) 4 × 5	22) $8\overline{)32}$	23) 3 × 6	24) 310 + 202 + 378
25) 12 − 11	26) 82 − 74	27) 7 × 2	28) $3\overline{)45}$
29) 370 + 86	30) 149 − 41	31) 2 + 50 + 6	32) 628 − 34
33) 6 + 1 + 5	34) 269 + 120	35) 12 × 4	36) 432 − 424
37) 676 − 622			

Student: Teacher: Date:

Reminder: There is **no** partial credit when scoring. The answer must be correct **in its entirety** to obtain the correct score value. Note that the answer key provides the correct answers for each problem, but is not exhaustive. For more information and examples of alternative correct answers, please refer to the Scoring section of the M–COMP manual.

Grade 3, Probe 25 Answer Key

Item No.	Answer	Correct	Incorrect	Item No.	Answer	Correct	Incorrect
1.	6	1	0	20.	4	2	0
2.	15	1	0	21.	20	1	0
3.	1	1	0	22.	4	3	0
4.	18	2	0	23.	18	2	0
5.	9	1	0	24.	890	2	0
6.	13	1	0	25.	1	2	0
7.	10	2	0	26.	8	3	0
8.	24	1	0	27.	14	3	0
9.	36	1	0	28.	15	2	0
10.	12	1	0	29.	456	2	0
11.	3	2	0	30.	108	2	0
12.	48	1	0	31.	58	1	0
13.	31	2	0	32.	594	3	0
14.	63	1	0	33.	12	1	0
15.	8	2	0	34.	389	2	0
16.	94	2	0	35.	48	3	0
17.	4	2	0	36.	8	3	0
18.	2	3	0	37.	54	2	0
19.	6	2	0				

Subtotal 1 ☐ Subtotal 2 ☐

TOTAL = Subtotal 1 + Subtotal 2 ☐

Appendix G

Math Journal

Numbers of the Week:

____ and ____

Tuesday:

Show three different ways to represent each number using golden bead or stamp game material

Wednesday:

a) write each number in expanded form.

Example: 47=40+7, 234=200+30+4

b) find the sum of two numbers. Explain your strategy. Show how you can use a different strategy to check your work.

Thursday:

Find the difference between the two numbers. Explain your strategy. Show how you can use a different strategy to check your work.

Friday:

Write and solve one addition and one subtraction problem using this week's numbers. See the back of this sheet for examples of different kinds of problems.

Monday:

Write three interesting facts about **each** number. For example:

- Is it an odd or even number? How do you know?
- How far is this number away from 10?
- How far is this number from 100?
- Can you count to this number when you count by 3's, 4's, 5's, 10's?
- How could you represent this number with money?

Adapted from K-5 MathTeachingResources.com

Add To

Result Unknown

15 birds sat in a tree. 21 more birds flew into the tree. How many birds were in the tree?

15+21=? *birds*

Change Unknown

15 birds were sitting in a tree. Some more birds flew into the tree. Then there were 21 birds sitting in the tree. How many more birds flew into the tree? 15+?=21 *birds*

Start Unknown

Some birds sat in a tree. 15 more birds joined them. Then there were 21 birds in the tree. How many birds sat in the tree to begin with? ?+15=21 *birds*

Subtract From

Result Unknown

21 birds sat in a tree. 15 birds flew away. How many birds were left in the tree?

21-15=? *birds*

Change Unknown

21 birds sat in a tree. Some flew away and 15 birds were left. How many birds flew away? 21-?=15 *birds*

Start Unknown

Some birds sit in a tree. 21 birds flew away leaving 15 birds in a tree. How many birds sat in the tree to start with?

?-21=15 *birds*

Adapted from K-5 MathTeachingResources.com

Appendix H

List of Children's Literature from Lynn Columba Article

Doubles facts:

Time for Bed (Fox 1997)

Two of Everything (Hong 1993).

Threes and fours facts:

A Three Hat Day (Geringer 1985)

Fives facts:

The Grouchy Lady Bug (Carle 1996)

Alexander, Who Used to Be Rich Last Sunday (Viorst 1978)

Zeroes and Ones facts:

The Principal's New Clothes (Calmenson 1989)

Squares:

Sea Squares (Hulme 1991)

Six facts:

Six-Dinner Sid (Moore 1991)

Nine facts:

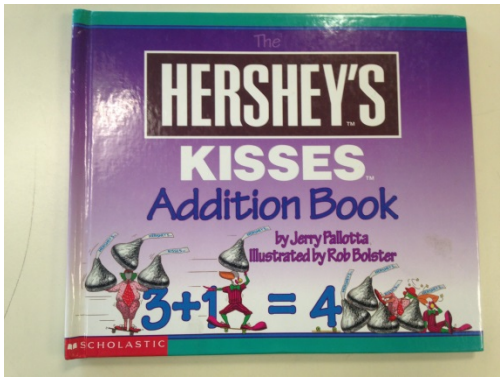
Finger Play from Arithme-Tickle: An Even Number of Odd Riddle-Rhymes (Lewis 2002 p. 16)

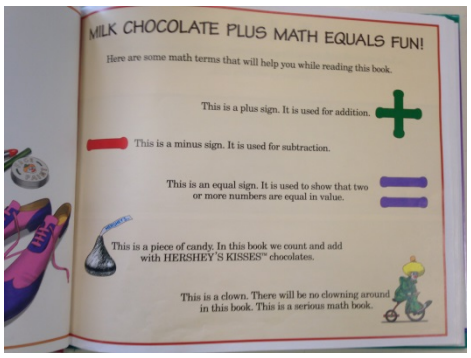
Remaining facts:

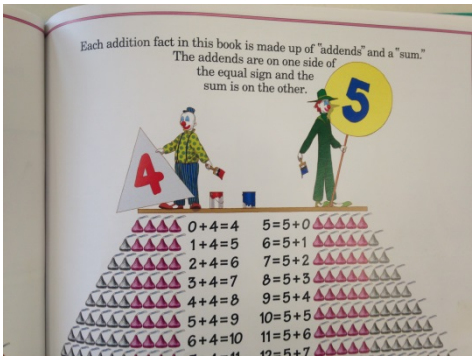
The Math Curse (Scieszka 1995)

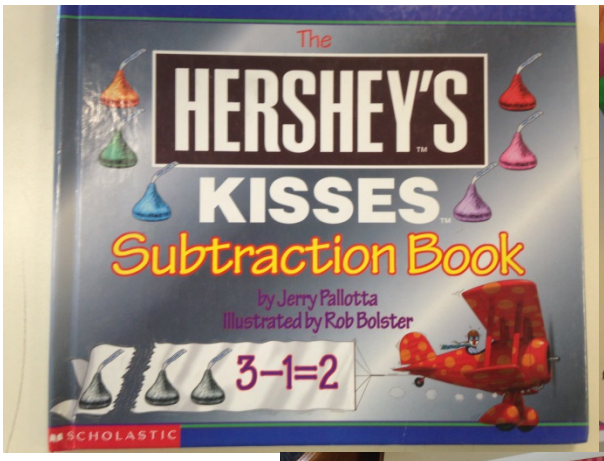
Appendix I

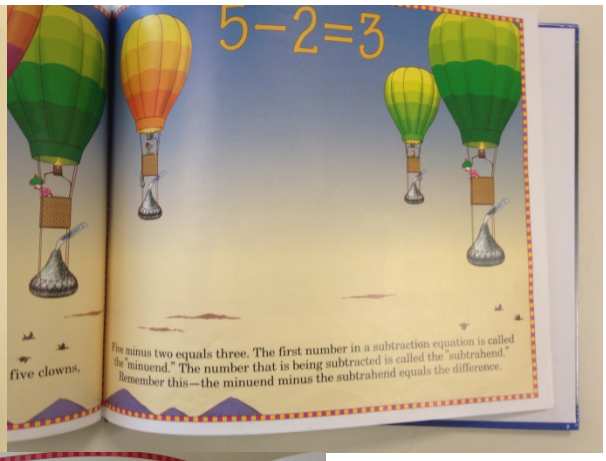
Books Used in Classroom for Enrichment of Concepts








The
HERSHEY'S
KISSES
Subtraction Book
by Jerry Pallotta
Illustrated by Rob Bolster
3–1=2
SCHOLASTIC


5–2=3
five clowns.
Five minus two equals three. The first number in a subtraction equation is called the "minuend." The number that is being subtracted is called the "subtrahend." Remember this—the minuend minus the subtrahend equals the difference.


6 feet
6 minuend
–4 subtrahend
2 difference or remainder
4 feet
When you do subtraction without taking anything away, it is called "comp
subtraction." The yellow clown is six feet tall. The blue clown is four feet
What is the difference in their height? Six minus four equals two.
The yellow clown is two feet taller than the blue clown.
300
The stripe
The gree

Appendix J

Math Vocabulary and Symbol Pre and Post Test

9+3=12

9-3=6

9x3=27

9÷3=3

Math Vocabulary and Symbol Pre and Post Test

A. What does this (+) mean?
B. What does this (=) mean?
C. Can you show me the addends?
D. Can you show me the sum?

A. What does this (-) mean?
B. Can you show me the minuend?
C. Can you show me the subtrahend?
D. Can you show me the difference?

A. What does this (x) mean?
B. Can you show me the multiplicand?
C. Can you show me the multiplier?
D. Can you show me the product?

A. What does this (÷) mean?
B. Can you show me the dividend?
C. Can you show me the divisor?
D. Can you show me the quotient?

Appendix K

November 1, 2014

Dear families,

In my pursuit of continuing education and Montessori certification, I am currently pursuing my Master's Degree in Montessori Education. Part of that pursuit includes conducting action research in our classroom. In conducting this research, I will analyze results of the strategies I use in class to support children and their mathematical thinking and problem solving skills.

I have observed our classroom community and have found a need to build math vocabulary and recognition of the type of math operation required to solve designated problems. To build vocabulary, my plan is to teach the children key vocabulary and symbol recognition needed to solve math problems. I will also be encouraging students to practice their new math vocabulary and recognition skills through the use of peer teaching, journaling and talking about their mathematical thinking.

I would appreciate having your permission to record your child during a class meeting and use my observations to support mathematical vocabulary and operations recognition to support mathematical thinking. I will use a pseudonym to protect your child's privacy. The information will only be shared with participants and in educational settings including a university mini-conference for teachers as a part of my research findings.

Please sign and return this form to me as soon as you are able. Thank you for your consideration and so you know, participation in this project is voluntary. If you have any questions or concerns, please email or call me at:

karen.olson@rfsd.k12.wi.us
715-425-7645, ext. 2232

Sincerely,
Karen Olson

I give permission for my child _____ to participate in the Building Math Vocabulary and Math Operations Sense Research Project during the 2014-2015 school year.

Parent's Name _____

Parent's Signature _____ Date _____

Child's Signature _____

References

Bay-Williams, J., Livers, S. (2009) Supporting MATH Vocabulary Acquisition. *Teaching Children Mathematics* Vol. 16 (4) 238-245

Boaler, J. (2015) Fluency without Fear: Research Evidence on the Best Ways to Learn Math Facts. *Retrieved at youcubed.org/fluency-without-fear*

Brett, A., Rothlein, L., Hurley, M. (1996) Vocabulary Acquisition from Listening to Stories and Explanations of Target Words. *The Elementary School Journal* Vol. 96 (4) 415-422

Change the Equation. org website. In a New Survey, Americans say, "We're Not Good At Math". 25 November, 2014. http://changetheequation.org/press/new-survey-americans-say-we're-not-good-math

Columba, L. (2013). So, here's The Story. *National Council of Teachers of Mathematics* Vol. 19 (6) 374-381

Dohrer, M. (2014). Adaptation of the Four Strange Brothers told at the UW River Falls Montessori Storytelling Workshop February 2014 (adapted by M. Lockwood)

Gay, A.S. & White S. (2002) Teaching Vocabulary to Communicate Mathematically. *Middle School Journal* Vol. 34 (2) 33-38

Gray, E., Tall, D. (1994) Duality, Ambiguity, and Flexibility: A "Proceptual" View of Simple Arithmetic. *Journal for Research in Mathematics Education* Vol. 25 (2) 116-140

Hulac, D., Dejong, K., Benson, N. (2012) Can Students Run Their Own Interventions?: A Self-Administered Math Fluency Intervention. *Psychology in the Schools* Vol. 49 (6).526-538

K-5 Math Teaching Resources website. 2nd Grade Number Activities. 30 October 2014 http://www.k-5mathteachingresources.com/support-files/2nd-grade-numbers-of-the-week.pdf

Kostos, K. & Shin E. (2010) Using Math Journal to Enhance Second Graders' Communication and Mathematical Thinking. *Early Childhood Education Journal* ,Vol. 38, 223-231

May, L. (1995) Speaking of Math. *Teaching PreK-8* Vol. 26 (3) p. 24-25

Montessori, M. (1989) *To Educate the Human Potential*. Oxford: Clio Montessori.

Moore, C. (1995) Math Assessment: Teaching the Language. *Teaching PreK-8* Vol. 25 (4), p. 50-51

Orr, J., Suh, J. (2013) A Picture is Worth 100 Math Ideas. *Teaching Children Mathematics* Vol. 19 (7) p. 458-460

Palotta, J. (2001)*The Hershey's Kisses Addition Book.* Illus. by Rob Bolster, Scholastic

Palotta, J. (2002) *The Hershey's Kisses Subtraction Book* Illus. by Rob Bolster, Scholastic

Petrill, S., Logan, J., Hart, S., Vincent, P., Thompson, L., Kovas, Y., Plomin, R. (2012) Math Fluency Is Etiologically Distinct From Untimed Math Performance, Decoding Fluency, and Untimed Reading Performance: Evidence from a Twin Study. *Journal of Learning Disabilities* Vol. 45 (4) p. 371-381

Ramirez, G., Gunderson, E., Levine, S., Beilock, S. (2013) Math Anxiety, Working Memory, and Math Achievement in Early Elementary School. *Journal of Cognition and Development* Vol. 14 (2) p.187-202

Smith, A., Angotti, R. (2012) Why Are There So Many Words in Math?": Planning for Content-Area Vocabulary Instruction. *Voices from the Middle* Vol. 20 (1) p. 43-51

Van De Walle, J. (2004) Elementary and Middle School Mathematics Teaching Developmentally (5th ed.) Boston: Pearson Education, Inc.