DOES WORKING WITH SETS CONTRIBUTE TO CONSERVATION OF NUMBER FOR YOUNG CHILDREN?

A Thesis

Presented to

The Faculty of the School of Education

Saint Mary's College of California

In Partial Fulfillment

Of the Requirements for the Degree

Master of Arts

By

Tracy Crawford

September 2008

Abstract

Montessori education has been criticized for teaching advanced abstract mathematics to children that do not have the ability to conserve number. This research provided children with the opportunity to work with specific teaching materials designed to give experience in working with sets. An understanding of the invariance of quantity is necessary to the development of conservation of number. This study did not show that specific changes to the curriculum were able to accelerate conservation of number. It did document that unusually young children can show conservation of number and that conservation of number occurs in uneven development.

Acknowledgements

The assistance and guidance provided by my chair Dr. Rosemary Peterson and readers Dr. Laurie Edwards and Patricia Chambers are gratefully acknowledged. Dr. Peterson has clarified Piagetian issues and early childhood. Dr. Edwards has been my primary source of any information about mathematics, early childhood mathematics, research methodology and statistics. I have truly appreciated the support and friendship of Patricia Chambers through several years of classes while at St. Mary's College. Patricia Chambers verified the accuracy of all Montessori information. Thank you all for your time and availability as I worked on this research about children and conservation of number. I have learned a great deal from all of you.

Will love and gratitude, I thank my family for their time and patience over the last few years. Your support and help has been invaluable. We have all made sacrifices and adjustments. Thank you Jimi, Kaylynn and Alyssa.

Table of Contents

List of Tables

List of Figures

List of Appendixes

Chapter 1: Introduction

Conservation of number is a mathematical concept that was first identified by Jean Piaget in the mid twentieth century. It is the recognition by a young child that quantity does not change with physical rearrangement. The development of conservation of number is an important cognitive milestone in a child's development. Research has shown that young children utilize more complex mathematical strategies once conservation of number has developed (Lemoyne & Favreau, 1981).Although conservation of number is not easily identified without formal assessment, there is still ongoing debate about what mathematics should be taught to young children or if the educational curriculum of young children should be modified (Caddell, 1998; Chattin-McNichols, 1998, pp 114-115; McClintic, 1988).

Conservation of number develops naturally in children, but it has been shown that its development can be accelerated with specific training in an experimental context. Currently, there is no research that evaluates changes within the framework of the curriculum and its effect on conservation of number.

This thesis evaluates whether specific changes, in particular the addition of work with unequal sets, to the Montessori mathematical environment, can accelerate the development of conservation of number in a classroom context. This thesis is tested empirically and the results are analyzed against a control group. The results have implications both for Montessori mathematical curricula and for the larger questions of curriculum design.

Montessori Method

First used in Rome one hundred years ago, the Montessori Method of education was developed by Maria Montessori. The Montessori Method is an educational philosophy that is based on observation of the developmental needs of the individual child and child directed activities that meet those needs. The Association of Montessori Internationale summarizes Montessori philosophy as follows:

> The Montessori approach offers a broad vision of education as an aid to life. It is designed to help children with their task of inner construction as they grow from childhood to maturity. It succeeds because it draws its principles from the natural development of the child. Its flexibility provides a matrix within which each individual child's inner directives freely guide the child toward wholesome growth.
>
> Montessori classrooms provide a prepared environment where children are free to respond to their natural tendency to work. The children's innate passion for learning is encouraged by giving them opportunities to engage in spontaneous, purposeful activities with the guidance of a trained adult. Through their work, the children develop concentration and joyful self-discipline. Within a framework of order, the children progress at their own pace and rhythm, according to their individual capabilities. (Association Montessori Internationale, 2008)

Current estimates place the number of Montessori schools in the United States at over 5000 (Lillard & Else-Quest, 2006) and approximately 7000 worldwide (International Montessori Index, 2008).

The Montessori Method includes a carefully designed curriculum. It includes many specific materials that support the developing number abilities of preschoolers. Two important components of early number sense which are supported in the Montessori mathematics curriculum: seriation and one-to-one correspondence. Seriation is the ability to order materials according to differences in size, weight, color, or other structured attribute. “One to one correspondence” is when a child can count or match a set of objects with one cardinal number or object (or matching object) to each member of the set. Many practical life exercises emphasize sorting or classification. Many of the sensorial materials (Pink cubes, brown prisms, red rods, knobless cylinders, and knobbed cylinders) support the child’s developing seriation abilities. These materials are all various specific manipulatives designed to exercise the senses. They are graded in size or shape. For example the pink cubes are a series of pink cubes with dimensions of 1 cm to 10 cm.

The early mathematics materials such as the spindle boxes and the red and blue rods help children to develop one to one correspondence.

Figure 1: Red & Blue Rods

Figure 2: Spindle boxes

The red and blue rods are 10cm to 1m, changing color every 10 cm. Their purpose is to demonstrate numbers as a whole quantity. Introduced without printed (or symbolic numbers) and the child learns to understand quantity through counting and physical manipulation. When the child is familiar with most quantities, for example “four,” and can identify or count quantities, then written numbers are introduced to correspond to the physical quantities. Some programs increase the one-to-one correspondence by providing small objects, such as teddy bears, to align to the red and blue segments of the rod. Subsequently, they may be used for basic addition or combinations forming ten. The spindle boxes are a set of numbers 0 to 9 with 45 wooden spindles. They introduce the child to the concept of zero and the concept that each number is made of individually consistent units. The material helps to establish counting as well as one to one correspondence. Given the research (Henry, 1976) that children still consider length to be a salient attribute of number, the red and blue rods help children to coordinate the numerical aspect of quantity with perceptions.

In the Montessori mathematics curriculum, the materials are presented in concrete form rather than symbolic form initially. The child develops an understanding of physical quantity before working with the associated symbolic numbers. The child only moves gradually to an increasingly symbolic and abstract level after substantial work with concrete materials.

Problem Statement

Children in a Montessori classroom work independently with a wide variety of math manipulatives as they move from concrete to abstract understanding. The question remains- are children learning how to successfully manipulate the mathematical materials without a deeper understanding of the mathematical concepts involved (Chattin-McNichols, 1998, p. 114) or working at individual levels of understanding and abstraction? Kamii's (2001) research indicates that children are able to work at different levels of abstraction, using the individual representation that is most suited to them, depending on the task. Research has shown children adapt a variety of strategies that adults do not anticipate to solve problems. (Fuson, Pergament, Lyons, & Hall, 1985)

Although there has been a great deal of research on conservation in general (Bryant, 1972; Field, 1981; Gelman, 1969; Rose, 1973) the studies involve specific training methods within an experimental context, rather than the development of conservation of number within the classroom environment. There is no research that examines whether the Montessori environment contributes to specific mathematical abilities or conservation of number at the preschool level. Therefore, it is not known whether providing students the opportunity to work on a component of number conservation, the comparison of sets, within the setting of an actual Montessori classroom, might be effective in scaffolding the transition from non-conserver to conserver. This research is designed to answer the question “Does experience with sets in a Montessori classroom environment help children develop conservation of number?”

Study Purpose

The purpose of this research is to evaluate whether preschool children in the preoperational stage of development (see below, Theoretical Rationale) are capable of increasing their development of conservation of number when provided with specific educational materials. The classroom material the children worked with was designed to help students understand comparisons of quantity. It consisted of two sets of pictures, a supporting grid framework, written numbers and vocabulary cards. This instructional material was consistent with other Montessori classroom jobs. While the material was available in the classroom, the quantities and pictures were varied twice a week. Instruction was provided in the correct use of the terms “more”, “less” and “same,” when first demonstrating the material to the children and individual lessons. The correct vocabulary provided additional experience for those children developmentally ready to learn the terms. This research proposes to evaluate whether preschool children showed changes in development of conservation of number after practical experience with sets for one month.

Theoretical Rationale

Jean Piaget developed a stage theory of the development of children based on his work and observations with children. Children move through several different stages of development- sensorimotor, preoperational, concrete operations and formal operations. Each is characterized by a different method of learning and different thought patterns. (See Theoretical Rationale, chapter 2- for additional detail). The preschool aged child is in the developmental stage called "preoperational." The children in this stage have been described as having "magical thinking" (Zusne & Jones, 1989) and lack logical reasoning. It is the preoperational stage where an absence of conservation of number has been observed.

Piaget's developmental theory led to substantial questions about conservation and related research about the development of conservation of number. Despite increasing knowledge about the developmental relationship of general number skills to conservation of number and evidence young children can be taught conservation of number, there is continued debate over the content and curriculum of mathematics education to young children (Caddell, 1998; Chattin-McNichols, 1998; Kamii, 1988; McClintic, 1988). One researcher (Gelman 1979; Gelman & Gallistel, 1986) in early childhood mathematics has said that we cannot compare the complete number abilities of the older child to the unknown but developing abilities of the preschooler and we should provide them with every possible opportunity to learn.

Method

Description of Research Design. This research was a quasi-experiment with two different convenience samples. Children in a Montessori classroom have traditionally chosen the materials that they wished to work with independently of the Montessori guide (or teacher). The variations that have developed over time in how the Montessori method is implemented have modified this practice at times, and in this study, the teacher/researcher presented an activity directly to the students, who could later choose whether to work more with the materials on their own. This research provided children who had no formal experience working with unequal sets an opportunity to explore the properties that arise when comparing sets. The classroom educational material was designed to support counting the individual members of each set, comparisons and development of the vocabulary of “more”, “less” and “same”. Each child was able to work with the material freely and at an appropriate individual level.

All of the children who participated were evaluated with a pre-test to determine initial abilities in working with number. This assessment included the ability to count orally (rote counting), and to physically count objects. Children were also assessed on whether they demonstrated conservation of number at the beginning of the study. At the end of the research study these same abilities were then reassessed.

Participants. This study was conducted at two different Montessori schools in the San Francisco Bay Area. The researcher taught at one school during the course of the study and was familiar with the director and staff of the second school. Both classrooms consist of children between the ages of two and a half and six years old. Socioeconomic status was not examined in detail as the general student population at both schools is predominately middle class. At the first school, the research was conducted in one classroom while the second classroom functioned as a control group. At the second school, only one class participated and all children participated in the research as an experimental group. There was not a second preschool classroom at this school. The first school was selected due to the researcher's ability to work with these children as a teacher in the environment. The second school volunteered to participate in this research after hearing of the project from the researcher.

Procedure. Children were given a simple mathematical assessment that included rote counting, counting out ten objects, counting a set of objects and conservation of number. Children in the experimental classrooms were given a demonstration of appropriate use of the educational material using standard Montessori instruction methods. The educational material was then placed in the classroom for the children to use for one month. At the end of the month all children were given the identical mathematical assessment.

The instructional material to introduce experience with sets included three containers with different quantities, the numbers one through ten, the words "more", "less" and "same" and a 2x10 grid that the children used to place individual objects into (Thompson, 1995). The material and quantities in the containers were changed twice weekly.

Data Analysis. The variable of interest was whether a child conserved number. It was measured by the number of children in each class who demonstrated conservation of number at the beginning of the study and one month later (after the experimental group received the new materials). A chi square analysis was used to determine whether there was a statistically significant increase in the proportion of conservers in the experimental group.

The children were classified at one of four levels of conservation of number *(Kamii, 2000)* based on their pre-test scores. After working with the educational material, analysis was conducted to see if the children showed signs of changes in their conservation of number development. Additional statistical analysis was conducted to determine any significant relationship to age. The remaining mathematical assessment was used as background knowledge for each child. It also served to relax the child prior to the conservation of number assessment.

Limitations

This research study has substantial limitations. These include limitations related to the study design, the natural development of conservation of number, use of the material, and differences in classroom instruction. There are criticisms of the traditional Piaget test for conservation of number and the way it is evaluated in young children.

The population was a convenience sample consisting of a final population from both schools of 57 children. The sample size is too small to be generalized beyond the children participating. The number of participants in the research may be insignificant to show statistical benefits with the research.

Children in a Montessori environment are free to choose their own materials, while teachers continuously provide individual educational lessons. Some of these children may find the material interesting and engaging, working with it extensively.

Others may have found it outside of their current interest range or unappealing. Because of the nature and routines of the classrooms, there is no practical method of documenting actual usage of the material by each child. An insufficient amount of time spent working with the educational material may lead to insignificant developmental benefits.

This experiment took place over the course of one month, during the age range when children may begin to develop conservation without any intervention or instruction. Thus it is possible to attribute any progress that the children have shown as a result of this experiment to the natural development of the children over time. Some children may have naturally developed conservation of number during this time period, and some children may be too young to benefit from the experimental material in a way that shows immediate development of conservation of number. A control group was used in order to determine the extent to which children of a similar age and background develop number conservation without the intervention. Natural development of conservation in the experimental group, unrelated to the intervention, may also exist. There is no way to determine the number of children that might have made this transition on their own.

The teaching material is flexible in design. Children may use the material in unexpected and unpredictable ways including comparing objects in a set in a non-linear manner. Some usage may reinforce length cues inherent in perceptions as unequal sets are frequently arranged so that one set is longer than the other. Other ways that children use that material may be developmentally appropriate but irrelevant to the development of conservation of number. Although designed to be used independently by the child, it is possible that interaction with a conserving individual provides the greatest developmental benefit with the material.

This study compared children in three different Montessori classrooms. The classroom structure is similar, but not identical in all environments. There were different teachers with individual teaching styles and personalities in each classroom. The differences in teaching styles or classrooms may affect the children's development of skills in general. It is not possible within the current research design to determine whether the differences in teaching styles were significant in relationship to the development of conservation of number. Some of the classrooms may have students that are used to working with new materials placed in the classroom or in the mathematics area. Other classrooms may have a structure where the children are not used to working in mathematics. This can significantly influence the use of the material.

When assessments for conservation of number are conducted in a different manner, children often perform with different results (See Literature Review). The performance of young children on a developing skill may vary daily and is difficult to assess with a single assessment at a point in time. Either the pre-assessment or the post-assessment may not be a true representation of the child's individual knowledge.

Assumptions

This study assumed that most, if not all, of the children in the preschool classroom are non-conservers according to the traditional Piaget assessment procedure at the beginning of the research project. It also assumes that there will be a variety of interest, attentiveness and motivation to work with the instructional materials. The pictures selected were done so with this in mind. Some of the pictures were specifically selected to be attractive to males, others to females and still others were designed to be appealing to all preschoolers.

Operational Definition of Terms

Conservation of Number – The ability of a child to recognize that the numerical value of a set does not change when the presentation of the material itself changes. Four levels of conservation have been recognized. In the first level, the child is unable to make an equivalent set. At the second level the child can create an equivalent set, but is unable to maintain the equality in the face of a perceptual change. At the third level, the child is unsure of equality or cannot provide a reason. At the fourth level, the child recognizes that the quantity does not change and can provide arguments to substantiate his reasoning (Kamii, 2000).

One to One Correspondence – The ability to recognize that each member of a given set has only one unique match. This can be object matched to another object such as a set of dolls matched to a set of doll beds. It can also be seen when counting a set of objects. When you count objects without "re-counting" you have automatically recognized the one to one numerical correspondence.

The Montessori Method- Developed by Maria Montessori, the Montessori Method is an educational philosophy that is based on observation of the developmental needs of the individual child and child directed activities that meet those needs.

Indirect Preparation – A Montessori concept where a particular material or lesson serves to also indirectly prepare the child for work that requires either greater physical coordination or mental preparation. For example, many of the activities in the practical life area work to build the fine motor skills of a young child, develop coordination of movement, concentration and self discipline. All of these activities are indirect preparation for writing.

Implications

Both Maria Montessori and Jean Piaget developed their theories of child development and education based on extensive observation of children. Piaget's theories of conservation of number have been extensively studied since its development. It is still debated what mathematical knowledge children learn prior to the development of conservation of number. The lack of knowledge about exactly what young children understand or can learn without conservation of knowledge affects everything from standard development, curriculum design to teacher education programs. If this type of intervention proves to be effective in scaffolding development of conservation of number in young children, it could be incorporated into early childhood educational programs. This research demonstrated that additional longer term research is necessary to determine the effect of modifications to curriculum and the relationship to conservation of number.

Chapter 2: Literature Review

Amidst increasing pressure to design and develop developmentally appropriate and academically challenging curriculum for preschools, while addressing state or national standards aimed at increasingly younger ages, there is a need to know more about how young children learn and what they know. This is true regardless of educational philosophy. This research evaluates if the development of conservation of number can be accelerated with changes to the Montessori mathematical curriculum.

This literature review describes Piaget's theory of Cognitive and additional research that refute some of the conclusions that Piaget reached. The last sixty years have seen substantial research on conservation of number. Many of the major studies are discussed in the section on Conservation of Number. Research that has specifically looked at the ability to induce or train conservation of number or its relationship to counting is covered in separate sections. The final section of this chapter is a discussion of the Montessori and Constructivist educational philosophies and related research.

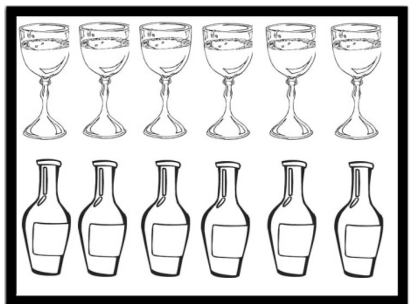
Theoretical Rationale

Piaget's Theory of Cognitive Development supports this research. Piaget believed that children construct their own knowledge of the world through interaction with the environment (Mooney, 2000). Piaget's research showed that all children move through four stages of development- sensorimotor intelligence, preoperational thought, concrete operations and formal operations. The sensorimotor phase is from birth to one and a half or two and is characterized by learning through the senses, reflexes and physical actions. The preoperational period, including preschool children, is typically defined from age two to seven. Piaget characterized preoperational thought as being egocentric, lacking scientific or logical reasoning, and developing the use of symbols- primarily language. Concrete operations, from age seven to eleven, is characterized by the child being able to think systematically but only with concrete objects while formal operations, from age eleven to adulthood, is characterized by abstract and logical thinking (Crain, 2005; Mooney, 2000).

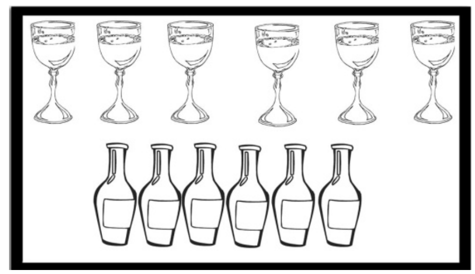
Jean Piaget (Piaget, 1965) conducted a number of experiments with young children where he determined that changes to the visual appearance of an object or group of objects influence the child's evaluation of quantifiable attributes, that is number or amount (e.g., volume or mass). All of these fell into the broad category of "conservation." A typical experiment is that the child agrees that the quantifiable attribute of two objects are equivalent. The researcher then changes the presentation of one object (a different container, spreading out objects, flattening an object) and again asks the child if the objects are equivalent. The non-conserving child does not find the second presentation equivalent. According to Piaget, a preoperational child does not remember the prior states and focuses overwhelmingly on perceptual attributes in their decision making process.

For example, Piaget (1965) initially placed six small bottles on the table and requested that the children get just the same amount of glasses. If the child placed either more or less, he was asked to help change them until the child was satisfied. Although Piaget does not explicitedly say so, it is assumed that he also then confirmed initial equality with the child before continuing. The length of the rows of either glasses or bottles was changed by spreading them further apart, and the children were asked again about equivalence. His experiments did not involve a script, but a conversation with each child that he worked with. Depending on their responses the children were categorized into developmental stages.

Figure 3: Representation of Piaget's initial conservation test with bottles and glasses.



Initial Presentation



Perceptual change

In the first stage, the children were unable to create exact correspondence or set equivalence. These children are still formulating basic numerical concepts. In the second stage, the children formed equal sets but when the appearance of one line was changed they believed that one element of the set had more. In the third stage, the children, despite the change in visual appearance, recognized that quantity had not changed, and were able to provide a justification for their evaluation. These children are said to have developed conservation of number.

Piaget demonstrated this result in a number of ways. Some examples include- beads poured from container to container, liquid poured from container to container, sets of objects including bottles and glasses, eggs and egg cups, flowers and flower vases, etc. that were placed first in matched rows and then with one row stretched out. Piaget felt that the closer the natural correspondence (flower and vase) between objects the child is more likely to notice inequality between the two sets, although additional research has not substantiated this assumption (Miller & West, 1976). The natural correspondence between two objects is called “provoked correspondence.” Jean Piaget found that conservation of number develops naturally and through the child’s own experience at approximately age six or seven (Price, 2004; Siegel & Goldstein, 1969). Conservation, in the Piagetian sense, is the ability to recognize that quantity does not change despite changes in appearance. Conservation applies to liquid, number, volume, length, mass, etc. Children may have difficulty with conservation of volume tasks up to age twelve. His work was so unusual that extensive research was done to substantiate his findings (Bryant, 1972; Gelman 1969; Mehler & Bever, 1967; Rothenberg & Courtney, 1969).

Lemoyne & Favreau (Lemoyne & Favreau, 1981) investigated if conservation of number affected the mathematical abilities of children when they looked at the operational strategies used in addition and subtraction. Children were separated into two distinct groups and classified as preoperational and operational based on tests in classification, seriation, conservation, and ordinal - number correspondence. These children were then given six types of problems involving addition and subtraction including missing addends. All of the children required assistance in learning strategies to solve five of the six types of problems. The strategies used in solving problems were then analyzed. The performance of the operational children was superior to that of the preoperational children. More of the children in the operational group were able to complete all of the problem types and mistakes were due to miscalculation or inattention. Preoperational children showed many of the same strategies but also relied on memory more frequently to solve problems.

> However, the fact that almost half the preoperational children not only performed well in addition and subtraction problems but also used strategies showing operational characteristics suggests that we should question the relevance of concrete operational thought as a competence factor related to mathematical learning.
> (Lemoyne & Favreau, 1981, p. 195)

This research showed that preoperational children are capable of substantially more complex strategies then previous research indicated. However, the superior performance of the operational group demonstrated that reaching the developmental milestone of conservation of number does affect cognitive development and performance. (Lemoyne & Favreau, 1981)

Egocentrism is the inability to see another's perspective. According to Piaget, this is a fundamental characteristic of preoperational children's thought and their lack of ability to reason logically. It implies that the child sees the world in the current moment or only in a series of disconnected images without relating one state to another. The child's inability to focus on the change in state leads to decisions based purely on perceptual decisions and contributes to a child's inability to conserve.

Piaget's conclusions regarding egocentrism were partially from experiments showing that young children were unable to identify the viewpoint of a doll when shown a model of three mountains. Martin Hughes worked with a variation of the viewpoint experiment that involved a constructed maze, and both policeman and child dolls (Donaldson, 1978). In this research situation, where children clearly understood the context including possible motives, even very young children demonstrated high accuracy. Does the behavior of children in the mountain viewpoint experiment reflect their lack of understanding of what is expected, communication issues, or attention to irrelevant stimuli? Gelman & Gallistel (1986) showed that speech patterns of a four year old when talking about a toy to a two-year-old child and to an adult are noticably different, reflecting the child's understanding of their audience. The research done since Piaget's initial studies indicated that the young child may be less egocentric and more capable of logical thought than Piaget's theory of child development suggested. Piaget's conclusions may have been influenced by his research design and the child's interaction with the situation. Donaldson's (1978) research indicated that children tended to reinterpret questions in individual meaningful ways when unsure of what is being asked of them.

Piaget believed that a child constructs number simultaneously with the development of logic and logical thinking (Piaget, 1965). Preoperational children will have difficulty in distinguishing part-whole relationships, class inclusion, classification and rely on their own perceptions in decision making. For example, a classic class inclusion example is to show the child eighteen brown beads and two white beads and ask them, "Are there more wooden beads or more brown beads?" Most preoperational children would say there are more brown beads (Piaget, 1965). Subsequent research has shown that Piaget's conclusions about young children's thinking may be inaccurate or incomplete.

Subtle changes in research methodology have led to significantly different research results (Donaldson, 1978). The class inclusion research is one example. James McGarrigle conducted an experiment with toy cows laying on their side. McGarrigle used two different questions to evaluate the child's understanding of class inclusion. "1. Are there more black cows or more cows? (the standard Piagetian form); and 2. Are there more black cows or more sleeping cows?" (Donaldson, 1978, p. 40) A statistically significant number of children were able to respond correctly to "sleeping cows" while responding incorrectly to the question phrased with "more cows" (Donaldson, 1978, p. 40).

Piaget's assertion that young children do not have the ability to make transitive inferences has also been reassessed. Piaget had concluded that preoperational children are unable to make inferences when certain information is available. (For example with the knowledge that A>B and B>C, children cannot deduce that A>C) Bryant & Trabasso (1971) determined that four year old children are capable of making transitive inferences provided care is taken to ensure that they can remember the individual pieces of information. Their study was conducted using colored sticks of different lengths and asking the children to compare them.

The recognition of a more complex pattern of the development of logical thinking and thought patterns of preoperational children has emerged from the research subsequent to Piaget.

Conservation of Number

An extremely comprehensive study on the elements and conditions that affected the development of conservation of number was conducted by Zimilies (1965). These included the ability of the child to delay gratification which appeared to be related to both conservation abilities and general developmental maturation. He also noted that the interest of the testing stimulus seemed to influence the children, although his own study reported inconsistencies. The trucks that he used did not seem to have been compelling to the children when they were not going to be able to keep them, and there was indication that the blocks which were meant to be a neutral condition aroused considerable interest. Zimilies also indicated that if young children are prevented from counting, or do not closely observe the transformation they may believe that the rearrangement actually involves adding or removing objects. This makes an accurate judgment difficult for the child.

Zimilies (1965) found that the performance of disadvantaged children compared to two other groups of children on conservation problems was both low and inconsistent. He looked at a low socioeconomic group of children compared to Jewish parochial school children. The early exposure to a formal curriculum and a variety of mathematical experiences may partially the differences in performance.

Bryant (1972) concluded that children as young as three are cognizant of the unchanging nature of number despite perceptual changes. His research showed that young children focus on several different "clues" and have difficulty in distinguishing the most salient feature of number – quantity. His research involved presenting children with two rows of objects that were unequal in quantity to determine whether the children could determine equality. In pilot he found that a set size of eight the children would provide answers that did not indicate random responses, and therefore he used a set size of 20 for the research experiment. The pilot studies were also used to determine the placement of the objects in the rows.

In 1972, Bryant tested conservation under several conditions to eliminate the length cue. From pilot studies he utilized three display situations. In the first, the rows showed visual correspondence and it was visually possible to identify the row that did not have a matched pair. Statistically children identified the more numerous row correctly more times than chance would predict. In the second situation, the less numerous row was spread out while the objects in the opposite row were very compact. Accurate determinations of “more” were statistically below chance. In the third situation, part of each line was closely spaced while the opposing objects were widely spaced. There were no visual clues for the child to make determinations of quantity and judgments were statistically equivalent to the rate of chance. The transformations involved moving the display from one to another. Bryant was looking for evidence that the child understood the unchanging nature of number if his decisions remained consistent after the transformation. He found that the response of three year olds in correctly identifying which row had more counters was statistically above chance. He concluded that young children are capable of conservation, but that they cannot easily distinguish between the reliability of length cues compared to visual pairs (one to one correspondence)

Bryant (1972) showed that some children had developed conservation of number unusually early, but his results were unable to be replicated (Sophian, 1995). The research by Katz & Beilin (1976) was similar to Bryant’s work, but their conclusion was that children were relying primarily on length cues. Katz & Beilin also reported that color and position were affecting the children’s judgments. Mehler & Bever (Mehler & Bever, 1967) also concluded that young children (2 to 3 years, 2 months) were able to discriminate based on quantity. Once again, subsequent research (Willoughby & Trachy, 1971) was unable to substantiate early conservation abilities.

Bryant's research has been criticized for potentially generating false positive results. His research pointed out potential flaws in the original methodology of Piaget including the length cue. He focused strictly on the principle that children recognize the unchanging nature of number.

Halford & Boyle (1985) conducted an experiment similar to Bryant's (1972) research. Halford & Boyle wanted to eliminate any possible false positives that might have existed in Bryant's research. Their hypothesis was that if children understood the invariance of number, judgments of quantity without counting would be consistent across transformations. Their experiment was similar to Bryant's (1972) except that they used four displays of the same length that pilot data showed produced neutral results. For all presentations there was no visual correspondence between the rows, some of the objects were compact while other sections were widely spaced. Transformations were from one neutral display to another. They concluded that three and four year olds do not understand conservation of number. Halford & Boyle do indicate that young children may be less sure of their judgments and therefore prone to changing their minds. This study used a large set size, prevented the child from counting, and eliminated both length cues and visual pairing with one to one correspondence. Understanding of invariance was only evaluated based on the child's initial commitment to which row had more across the transformation.

Starkey (1981) showed that children were more likely to make the correct response when the row with the greater quantity was transformed first. Starkey's results may have shown an unintentional methodology flaw in Bryant's work by determining that the row that was transformed first influenced the judgment of the children in determining quantity (Halford & Boyle, 1985). Both of these studies (Katz & Beilin, 1976; Starkey, 1981) have been criticized on the grounds that the distinction between pre-transformation and post-transformation may not have been distinct to the child (Sophian, 1995).

For some time, one of the most reliable experimental findings in cognitive development has been that preschool children under the age of five will fail the standard Piagetian number-conservation test (Gelman & Gallistel, 1986). The failure of young children to successfully pass the Piagetian test for conservation of number may be due to several factors. Issues as simple as the comprehension of the language used, interest in the materials, distractibility and memory demands can influence the performance of preschoolers (Miller, Heldmeyer, & Miller, 1975).

Rose (1973) found that language is particularly important when evaluating young children for conservation of number. When three and four year-olds were presented with both equal and unequal sets and asked if there were the same amount of objects, children tended to respond in the affirmative. Rose & Blank (1974) found that asking only one question after the transformation of objects increased conservation responses in six year old children. This finding was documented with a wider age range of children by Samuel & Bryant (1984). They found that children between 5 and 8 ½ showed similar results on tests for conservation of number, mass and volume. Their conclusion is that repetition of the question causes children to question their own judgement and change their response. She also showed that young children have difficulty with the verbal explanations that are required to document complete number conservation and often merely agree with the examiner (Rose S. A., 1973).

In the traditional Piaget test for conservation, children must attend to quantity, length and visual clues of correspondence. Transformation in the traditional Piaget test for conservation of number may draw the child's attention to changing the visual stimulus while quantity is static. Gelman (1969) and Zimilies (1965) have shown that young children can attend to shape, spacing or other attractive stimulus. Gelman showed that with extensive examples of training problems with a variety of conservation problems, children learned conservation of number and frequently generalized their conservation abilities (Gelman, 1969). This research focused specifically on the need to teach children to ignore irrelevant visual changes and length clues. Miller, Heldmeyer & Miller (1975) indicates that most assessments for conservation of number begin with two rows of equal length which might implicitly indicate the importance of length as a dimension of quantity to the young child.

Young children see number as being "multidimensional" (Gelman, 1969). Quantity is only one dimension and they have not yet learned to discriminate its importance. This is similar to a young child's representation of what a "dog" is. "Presumably our mental representation of the class 'dog' contains in some manner the attributes we expect to find in a dog, the sorts of things we expect him to do, and his likely relationship to places, people and other animals" (Macnamara, 1975, p. 424). Yet the ability to accurately define a dog or a chair can be elusive. An adult thinks of different animals when thinking of a dog and a wolf due to different behavior, different physical characteristics, different natural environments and heritage. A child when examining pictures of a wolf and a Siberian Husky would probably have more trouble in distinguishing the two animals without additional information.

Young children must learn the critical elements of number. To the young child, the attribute of quantity is only one aspect. The child must learn that other aspects are irrelevant, including density, spacing, color, length (Gelman & Gallistel, 1986). Henry (Henry, 1976) studied the initial attention preferences of both kindergarten conservers, non-conservers and third grader children. Prior to testing conservation he tested preferences using arrays where “three rows of poker chips were presented in such a way that a child could choose two the same on the basis of number, length, or density” (1976, p. 752). He found that 75% of the third-grader’s preferred number while only 26% of the kindergarten children preferred number. Sixty percent of the kindergartener’s initial preferences were based on length. Henry also found that children responded to a quality he called “gappiness” or the presence of a gap in the arrangement of a set will influence children’s judgments of quantity.

In 1970, research began to address the issue of whether various “perceptual” supports would help children show conservation of number. Whiteman & Peisach (1970) looked at guidelines, matching color, and training with reversibility as overall elements of conservation of number. For their research they specifically looked at either drawn lines between set elements as guidelines or elements aligned by color to each other. They noted improvements, especially in older children, in conservation results. Miller, Heldmeyer & Miller (1975) then followed up with research based partially on the work of Whiteman & Peisach. They concluded that the development of conservation of number involves several stages. Their research was specifically designed to identify conditions that facilitated conservation. Pretesting was specifically designed to familiarize the child with the type of question asked and ensure vocabulary knowledge. The order of questions was alternated between conservation trials, and the examiner probed the child's thinking with follow-up questions. The conservation trials included different set sizes and interesting stimuli. Rows were placed in lines vertical to the child's vision to avoid any tendency to choose the closest row as having more. These researchers had carefully looked at the previous research involving the tendency of young children to indicate that the closest horizontal line had a greater quantity and designed the conservation assessment to preclude this bias. One of the conservation trials included inequality to avoid a response bias of the child always identifying the sets as the "same." With these changes to the traditional Piaget conservation test, Miller, Heldmeyer & Miller (1975) found that 17 of the 20 three year olds showed evidence of conservation. Fifteen of these children supplied an adequate explanation for their judgments.

Rothenberg & Courtney (1969) found that young children were influenced in the development of conservation of number by several factors including length, density and proximity. They also found that socio-economic status affected understanding of vocabulary and therefore understanding of the problem.

Trainability of Conservation of Number

One research question that arose after the initial research on conservation was, "Can children be taught conservation earlier than it naturally develops?" In the sixties and seventies, there were a number of studies on training or teaching conservation of number. Most of these studies were with children closer to the traditional age that conservation of number naturally occurs. The conservation studies showed mixed results, and often were not able to show sustained conservation from the research experiment that did not revert over time. Research with preschool children in this time period involving teaching conservation of number was still largely unsuccessful.

In 1964, Wallach & Sprott (1964) conducted a training experiment which induced lasting conservation in first grade children by emphasizing reversibility. Children worked individually with an examiner to examine different arrangements of dolls and doll- beds. These included moving the dolls, the beds and the addition or removal of a doll or bed. This study was the first large study to demonstrate lasting conservation of number.

Then in 1969, Rothenberg & Orost were able to successfully train kindergarten students in conservation of number. The effects of their training were retained for at least three months. They conducted a series of three experiments with kindergarten students designed to teach the logical steps necessary for the development of conservation of number. One of the unique features of their study was that they utilized slightly older conserving peer instructors during one of the last training sessions. Rothenberg & Orost felt that conflict from opposing points of view from a child close in age would be an effective tool in changing cognitive structures. They also found that the peer instructors were able to communicate meaningfully with the other children.

Schneck & Canaday (1974) conducted an experiment where they tried to teach conservation of number to preschool children (average age- 4 ½ years old) with two approximately fifteen minute training session. Their intervention failed to show any differences between the experimental and control groups, and they concluded that direct instruction is ineffective.

In a structured training program of three, four and five year old children, Deal (1967) explored if the development of one to one correspondence was sufficient for the development of conservation of number. Children worked with a research assistant in small groups for sessions of eight days for approximately thirty minutes each day. His training program was effective in developing the concept of one to one correspondence, especially with the older children. Although the children learned the concept of one to one correspondence, it was insufficient to develop conservation of number skills at that time.

A review was conducted by Brainerd (1977) on studies that dealt with cognitive development and concept learning while Field (1981) conducted an extensive review of conservation studies that involving training. Brainerd determined that although there are fewer conservation studies of preschool children or children on the lower age range of preoperational behavior, these children can be taught conservation of number. Most training procedures will have a smaller effect than with older children, but preoperational children are capable of developing general conservation knowledge. Field concluded that "four year olds were able to generalize to untrained quantities far better than three year olds, however, and in a follow-up posttest given 2½ to 5 months later, those children that had generalized maintained or increased their conservation understanding, while almost all three year olds reverted to nonconservation" (Field, 1981, p. 2) . Field's own research shows that four year olds can be taught conservation but that any conservation that three year olds demonstrate is very short lived. Currently there is sufficient research that young children can be taught conservation. The differences exist in training methods and age of the children.

Conservation of Number- Counting

Rothenberg and Orost (1969) identified several steps that they assumed underlie the development of conservation of number. These include the following:

> (1) Rote counting, (2) counting attached to objects, (3) “same” number, (4) the “same” versus “more” distinction in terms of number, (5) addition and subtraction representing a change in number, (6) one to one correspondence, (7) reversibility and (8 the distinction of “more” referring to the actual number of objects versus “longer” referring to their arrangement in space. (Rothenberg & Orost, 1969, p. 710)

This research material designed for this project focuses on the first four steps identified by Rothenberg and Orost. It includes counting, counting objects and practical experience with objects in understanding the concepts of “same,” “less,” and “more.”

The interaction between counting, number relationships and conservation of number are complex and may develop simultaneously. Saxe (1979) studied counting skills in children between ages four and six years old and in learning disabled children between seven and nine years old. His research tried to determine the developmental sequence between the development of counting skills, counting strategies and conservation of number. Counting strategies include counting an array in an ordered or complete pattern. He concluded children develop counting skills, and may even count inaccurately before developing conservation of number. “Counting inaccurately” is when the child’s counting reflects omissions of numbers, an inaccurate order of cardinal numbers when counting or counting the same object more than once. Unlike Piaget, he believes that counting skills represent more than rote knowledge and contribute to the development of conservation of number. However, because counting accurately is not necessary for the development of conservation of number, it may reflect the child's development in understanding of one-to-one correspondence between a consistent or static array and dynamic arrays.

Catherine Sophian (1995) has shown that there is a developmental relationship between counting and the development of conservation of number, but that there is some evidence that conservation may precede counting. She performed an experiment where children (three to six) were given a total of 16 problems. Eight of these problems involved conservation tasks and eight of these problems involved substitutions. In each case there was a ribbon with fixed objects that was considered the stationary set. For the conservation and substitution set the objects were fixed to elastic that could be easily adjusted. In all cases the objects did not align. Children were classified as either "number" responders if they showed any evidence of counting or "length" responders if they choose the longest row on 13 of the 16 tasks. There were 21 consistent conservers in her sample, only ten of whom showed signs of consistently counting prior to the transformation. There were 36 children who were classified as non-conservers, only two of whom showed signs of counting prior to the transformation. She also performed similar experiments where only part of the display was shown at a time while counting was discouraged. To ensure that the elastic strips were not affecting the results with an unintentional length cue a third experiment was performed without elastic strips. The results on these experiments were similar to the first. Sophian did indicate that there was a close relationship between both conservation and counting to the child's chronological age. The six-year-old subjects showed greater differentiation when choosing to count in the substitution problems compared to the conservation problems.

Fuson, Secada & Hall (1983) examined children's spontaneous use of counting or matching strategies in conservation of number problems. Their research focused on two experiments with three different situations- a traditional Piagetian conservation of number task, a separate task where the children were reminded to count, and a matching task. In the matching task the two objects (zoo animal and peanut) were attached by a string. The order of tasks was varied in order to assess differences between control, matching and counting. When the traditional Piagetian assessment came first, only 2 of 14 children correctly determined equivalence, while 11 of 16 children determined equivalence when reminded to count and 12 of 12 were able to determine equivalence when given a matching strategy. However, substantially more children were able to provide verbal explanations related to counting than to matching. They concluded that most children between four and a half and five and a half have matching and counting skills sufficient to make equivalency determinations about two sets regardless of perceptual information, but that they do not spontaneously use these strategies. Slightly older children will begin to spontaneously count or use matching strategies.

Zhou (2002) also investigated children's use of counting to compare two different quantities. Although counting skills may be a prerequisite for comparison of sets, other researchers have disagreed with this assumption. However, lack of counting skills, lack of confidence in counting skills and lack of experience may contribute to children's failure to select counting as a strategy when comparing to quantities (Zhou, 2002). Young children may have found that perceptual cues which generally serve well in everyday life are more reliable than their developing counting skills as an effective strategy for comparison. Zhou's finding is consistent with that of other researchers that children do not spontaneously use counting (Fuson, Secada, & Hall, 1983).

The term "Subitizing" (Kaufman, Lord, Reese, & Volkmann, 1949) means to instantly recognize a quantity without counting. Subitizing can occur with three to seven objects and may be a relevant factor in research results that show conservation of number with unusually young children. In young children, counting accurately depends on the quantity they are being asked to enumerate.

Winer (1974) conducted research with sets of size two to three and sets of size five to six and found that there was a greater display of conservation in four year olds with the small set. This may be a perceptual decision due to subitizing rather than a true understanding of invariance. Zimiles (1965) also found that a small set size affected children in conservation experiments. He determined that when a small set was presented initially, children did better when subsequently presented with larger sets. Miller, Heldmeyer & Miller (1975) reports that studies that focused on set size have shown mixed results.

Gelman (1972) found that when three to six year olds were working with extremely small sets (two or three) they showed surprise when objects were surreptitiously added or removed. Her experiment was conducted under the guise of a magic experiment with "winners" and "losers." The children became involved in gaining prizes as they played the game and the examiner was able to add or subtract objects. Gelman said, "When the numbers are small, as they were here, children spontaneously focus on the numerosity of a set. As a consequence, they are more surprised by changes in numerosity than by changes in length and density." (Gelman, 1972, p. 88)

Many preschoolers have learned that in order to answer the question, "How many are there?" they use the last counted word as an effective strategy for the correct response. Fuson, Pergament, Lyons & Hall (1985) found that for extremely small sets young children often counted accurately, for sets between 4 and 7, equal numbers would demonstrate a last word response and accurate counting, while for sets from 9 to 19 most children gave a last word response.

Although rote counting and counting objects have been presumed to be prerequisites for the development of conservation of number, researchers have reached different conclusions about the relationship between accurate counting and conservation of number. The relationship between conservation of number and one to one correspondence has been more difficult to analyze.

Montessori and Constructivist Philosophy

Maria Montessori developed her philosophy of education a hundred years ago. The term "Montessori" is in the public domain and is not protected by trademark. As a result, there is a extensive variety in Montessori schools. Several professional organizations exist but, there is not a professional organization that maintains consistent oversight over all Montessori schools. As Daoust (2004, p. 37) said, "… any school, regardless of its practices, could call itself a Montessori program."

Daoust's research (2004) has shown that, in practice, Montessori philosophy has diverged into four distinct subcultures. She classifies these groups as "Traditional," "Contemporary," "Blended" and "Explorative." Each of these groups has different characteristics dependent on four key aspects of Montessori philosophy. These include mixed-age groupings, the length of the work period, whole group lessons, and the percentage of Montessori materials in the classroom (Daoust, 2004).

Jean Piaget (1896-1980) was familiar with the work of Maria Montessori (1870-1952). Both Jean Piaget and Maria Montessori developed educational theories that are based on the child developing or constructing their own knowledge through individual choices. He believed that the goal of education was to create independent critical thinkers capable of doing new things (Enotes.com, 2008; Kamii, 1988). "Constructivism" is a theory of development while "Constructivist" is the term that is given to schools that have a strong commitment to this philosophy in their curriculum. Kamii (1982) defines constructivist education, based on the work of Jean Piaget, as having a goal of moral and intellectual autonomy.

The Montessori and Constructivist philosophies are similar in many ways, yet significantly different in others. The philosophies are fundamentally different especially in the mathematics curriculum. The Montessori mathematics curriculum is very well defined and presented in a particular order. The Constructivist mathematics curriculum has a strong emphasis on games, and building relationships from daily tasks. The goal in a constructivist mathematics education is for the child to individually construct logico-mathematical knowledge. In the Montessori mathematics curriculum, the objective is to provide the child with a mental understanding of both the "whole" and the "individual parts" that make up a system of mathematics in a concrete way.

Research comparing the children from the different philosophies has been conflicting. Recent research by Lillard and Else-Quest choices (2006) has favored

Montessori students when evaluated on academic skills and social or behavioral. This research shows that when implementation of Montessori practices is consistent with the philosophy developed by Maria Montessori and as recognized by the US branch of the Association of Montessori Internationale (AMI/USA) five year olds showed significant differences in academic readiness while 12 year olds wrote significantly more creative essays with complex sentence structure. When tested on social behavior, the younger children were more likely to use higher levels of reason that included justice or fairness while the older children choose the most positive assertive response such as expressing feeling (Lillard & Else-Quest, 2006). This group would be similar to the traditional group as described by Daoust (2004). Older research has also show long term benefits to children in Montessori programs. "One long term study (Miller & Dyer, and follow-ups) found that Montessori children, particularly males, showed statically significant differences in math achievement test scores as late as tenth grade, in comparison to both other preschool programs and no-preschool control groups," (Chattin-McNichols, 1998, p. 114). Significant numbers of Montessori children had the ability to seriate and to classify objects compared to children in traditional preschools. (Yussen, Matthews, & Knight, 1980). The same study did not show differences in conservation abilities but the researchers attributed that to the more advanced nature of conservation. In contrast with the research of Lilliard & Else-Quest (2006), the research of DeVries and Goncu has shown that children in a constructivist program are more advanced on certain social and cognitive measures than children in a Montessori program (DeVries, 2002; DeVries & Goncu, 1968). This is partially attributable to the variety in implementation of Montessori practices that has developed over time. There have been a limited number of research studies concerning Montessori educational results. Additional research, documenting the mathematical abilities and understanding of children in Montessori classrooms is needed. Specifically, research about how young children's math skills develop in different environments is needed.

Conclusion

The accumulated research shows that four and five year olds can often be taught conservation of number. These skills are often generalized to other conservation attributes. Three year olds may sometimes acquire conservation skills depending on the training procedures and quantities, but it is less often sustained or generalized. However, the research has not been translated into a consistent curriculum for preschool children which helps them to develop conservation of number through experience.

The successful conservation training studies have led to many insights into how children develop number. Subsequent research has evaluated changes in methodology or assumptions. Some of the factors that have been identified as influencing the training of conservation of number include counting abilities, knowledge of strategies, feedback, type of pretest, and language skills. Some of the factors that have been identified as affecting research results include set size, questions asked, research design, placement of sets and transformations.

Chapter 3: Methodology

Purpose

Although there has been numerous research studies that have shown young children can be taught conservation of number, this research was specifically designed to evaluate if the development of conservation of number can be accelerated with minor curriculum changes in the classroom environment. The Montessori classroom is a child directed environment where the child is able to choose materials to work with. This research introduced a specially designed educational material that allowed children experience with working with sets. It provides addition evidence for the question, "Does working with sets contribute to conservation of number for young children?"

Study Design

The current study was designed to show if children's conservation of number abilities change specifically from experience with working with sets. Researchers (Miller, Heldmeyer, & Miller) noted in 1975 that "conservation" depends on procedures used and criteria for assessment. The conservation of number assessment used in this study was designed similar to Piaget's original procedures. This is considered the strictest possible assessment of conservation of number. An initial set size of eight was chosen because it was larger than the quantity generally considered possible for a child to subitize, yet small enough that a preschool child was not precluded from using counting skills.

This research was conducted as a quasi-experiment. The schools were chosen as convenience samples and the participants were selected based on pre-existing classroom groups. The research was designed to minimize interruptions to the natural classroom activities and environment. The assessments conducted at the beginning and end of the experiment regarding basic mathematical skills and conservation of number were the only activities that reflected a change to the child's normal routine.

Setting

This research took place at two Montessori preschools located in Palo Alto, CA and Redwood City, CA. Palo Alto is a middle class to upper middle class neighborhood in the heart of Silicon Valley. In 2006 and 2007, the median home price was more than $1.3 million (Coldwell Banker, 2007; see also Blitzer). The children that attend this preschool are representative of this high socio-economic background. Census data for Redwood City (Association of Bay Area Governments, 2003) indicated that in 2000 the median household income was $66,700 and that the median value of owner occupied housing was $517,800. Projections for 2007 indicated that median household income was expected to be $71,600 while the median value of owner occupied housing was expected to be $802,000.

The researcher worked with the children at the school in Palo Alto from several months to over a year depending on individual enrollment, but was not familiar with the children at the Redwood City school. The ability to conduct research in a natural teaching setting with familiar adults was a consideration in the research.

Participants

All families at both schools were invited to participate in the research study after approval was obtained by the Internal Review Board of St. Mary's College and the school director by letter. The subjects in this research study are predominately Caucasian. There is some minor ethnic variety in each classroom.

Eighty five percent (47 of 55) of the children's parents in Palo Alto consented to participate in the research study while 100% of the children in Redwood City consented (22 of 22). In Palo Alto, only 38 (69% of all students) of those children eligible to participate (80.85%) actually completed both assessments. There were 18 children in the control group and 20 children in the experimental group that completed the assessment. At Redwood City, 3 children were unable to participate in either the pre or post assessment (86.36% participation). The lack of participation was primarily due to refusal on the part of the child to participate in either the pre or post assessment.

At the school in Palo Alto, there were 21 subjects in the control classroom and 26 subjects in the experimental classroom. The subjects were between 2 ¾ and 6 years old at the time of the research. In the control classroom there were twelve 4-year olds and nine 5-year olds. The mean chronological age was 4:9. In the experimental classroom there were a 2-year old, four 3-year olds, nine 4-year olds and twelve 5-year olds. The mean chronological age was 4:7.

At the school in Redwood city, there were 22 subjects in the classroom. The subjects were between 3 and 5 ½ years old at the time of the research. There were seven 3-year olds, ten 4-year olds and five 5-year olds. The mean chronological age was 4:3. This group of children was younger than both groups at the school at Palo Alto.

Table 1: Participants Age

Palo Alto Classroom			
Control Classroom		Experimental Classroom	
Chronological Age	# of Students	Chronological Age	# of Students
2 ½ to 3	0	2 ½ to 3	1
3 to 3 ½	0	3 to 3 ½	0
3 ½ to 4	0	3 ½ to 4	4
4 to 4 ½	6	4 to 4 ½	6

4 ½ to 5	6	4 ½ to 5	3
5 to 5 ½	8	5 to 5 ½	11
5 ½ to 6	1	5 ½ to 6	1
Mean age	4.9	Mean age	4.7
Total Students	21	Total Students	26

Redwood City Classroom
Experimental Classroom

Chronological Age	# of Students
2 ½ to 3	0
3 to 3 ½	1
3 ½ to 4	6
4 to 4 ½	5
4 ½ to 5	5
5 to 5 ½	5
5 ½ to 6	0
Mean age	4.3
Total Students	22

At Palo Alto, there were 43% girls, 57% boys (9 girls / 12 boys) in the control classroom and 61% girls, 38 % boys (16 girls / 10 boys) in the experimental classroom. In Redwood City, there were there were 36% girls, 64% boys (8 girls / 14 boys) in the control classroom.

Table 2: Participants Gender

	Palo Alto- Control	Palo Alto- Experimental	Redwood City
Girls	9	16	8
Boys	12	10	14
Total	21	26	22

Measurements

Participating subjects were given a brief mathematical assessment to assess the following: rote counting, counting ten objects, counting a set of objects, and conservation of number. Each assessment was conducted individually with the researcher in an area of the school familiar to the children.

Rote counting. The child was asked, “Can you show me how high you can count?” If the child showed any sign of confusion the researcher said, “1, 2, 3…” to assist and support the child. The highest score that a child counted to without errors was recorded.

Counting Out Objects. One basket of fifteen wooden ‘suns’ was on the rug. The ‘sun’ has a diameter of three inches. The subject was asked, “Can you count out 10 ‘suns’ for me?” The child’s actions were observed and documented. A numerical score was given for the actual number of objects counted out by the child on a scale of 0-10.

Counting Set of Objects. The researcher placed a set of twenty small wooden musical treble clef symbols in a random pattern on the rug.

Figure 4: Wooden Sun

Figure 5: Musical symbol counted

Each child was asked, “Can you count these for me?” The set was reduced by six until the set was successfully and accurately counted. On subsequent sets, the child was asked, "Let's try these. Can you count these for me?" The child was given a score (30, 24, 18, 12, 6) based on the size of the set that they correctly counted.

Conservation of Number. The researcher then placed eight red poker chips down in a line with approximately equal space between them. The child was then asked, "Can you put out the same number of blue poker chips?" The child's responses were observed and recorded only.

The child was asked, "Are there the same amount of red chips as blue chips or are there more red or blue chips?" One line of poker chips was then spread out, and the child was asked, "Are there still the same amount of red chips as blue chips or are there more red or blue chips?" If the child replied, "the same," the researcher will ask, "Can you tell me why?" Responses were documented and categorized. Children were assigned one of four levels of conservation of number depending on their ability to recognize initial equality and maintain that recognition after spatial rearrangement.

Table 3: Conservation of Number

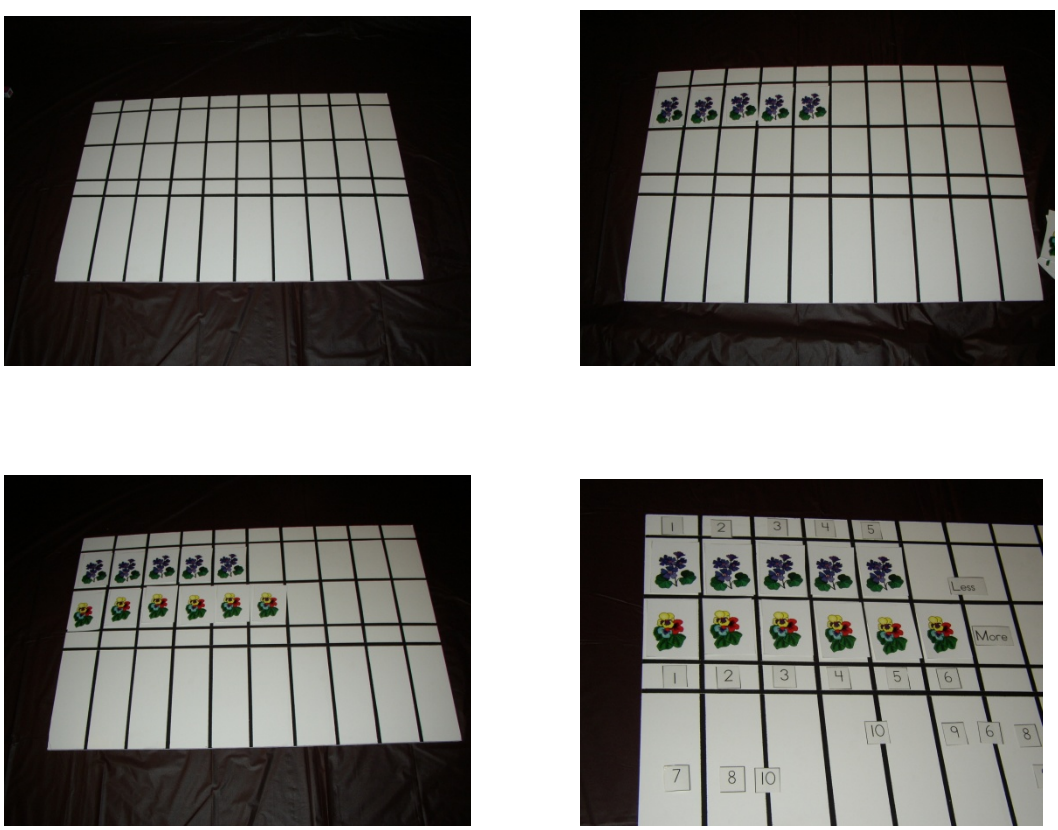
Level 0	The child does not recognize the initial equality of the sets.
Level 1	The child recognized the initial equality of the sets, but did not recognize equivalence after a perceptual transformation.
Level 2	The child recognized equivalence before and after the transformation but is unable to provide a reason or is unsure of their decision.
Level 3	The child recognizes the equivalence of the sets before and after the transformation and provides an adequate justification for their decision.

The justifications that a child might provide include reversibility, “You could put them back,” invariance, “You didn’t change anything,” compensation “You just spread the line out,” and identity, “You didn’t add any or remove any.” A child that can verbally justify his explanations is said to have completely developed conservation of number.

Procedures

Experimental Material. The experimental teaching material is a job designed in a Montessori fashion to be used by an individual child or small group of children. The material consists of a white foam core board (30 inches x 20 inches) which is marked into an 10x2 array.

Figure 6: Experimental Material



There were three sets of pictures available for the children's use (See Appendix A). Each picture was approximately 1 ½ by 2 inches. The pictures were printed on cardstock, cut, laminated and trimmed. The quantity and pictures were varied twice a week according to the following table. The material contained two sets of numerals for use in counting. The words "More," "Less" and two copies of "Same," were also part of the material.

Table 4: Palo Alto - dates for picture sets

Introduced	Picture & Quantity		Picture & Quantity		Picture & Quantity	
5/20/08	Scutosaurus	2	Diplodocus	3	Triceratop	5
5/22/08	Gray horse	4	Running horse	6	Standing horse	7
5/27/08	Abstract purple	2	Abstract olive	4	Abstract blue	6
5/29/08	Bulldozer	3	Digger	6	General construction	8
6/3/08	Green butterfly	4	Dark blue butterfly	7	Mixed blue butterfly	8
6/5/08	Abstract yellow w/ blue	5	Abstract yellow w/ red	6	Abstract purple	7

Table 5: Redwood City- dates for picture sets

Introduced	Picture & Quantity		Picture & Quantity		Picture & Quantity	
5/14/08	Standing lion	2	Relaxed lion	3	Zebra – 5	5
5/16/08	Gray horse	4	Running horse	6	Standing horse	7
5/20/08	Sea lion	2	Chipmunk	4	White dog	6
5/22/08	Green Butterfly	4	Dark blue butterfly	7	Mixed blue butterfly	8
5/27/08	Red car	6	Purple car	7	Green Truck	10
6/2/08	Pansies	4	Violets	7	Yellow Flowers	8
6/4/08	Blue boat	3	Square rigger	6	Black boat	9

Introduction of Teaching Material. Although Montessori philosophy advocates small group or individual lessons many schools have choosen to make group presentations. The children at both of these Montessori schools were used to having new classroom materials presented at a large group lesson. Therefore, the research material was initially presented to the children in the experimental classroom as a group lesson. Individual or small group lessons were given to those children not present for the initial lesson. Children in the experimental classroom were then free to choose to work with this material as desired. The children in the control classroom were not shown this material or given this lesson and continued to work normally. They were provided with this material at the end of the one month experimental period.

Montessori lessons are typically quiet with a minimum of words allowing the child to focus on the concrete materials. At the beginning of the lesson, a work rug was unrolled and the materials were carried from their position on the math shelves. The board was laid out. The first set of pictures was shown to the children and laid out in the grid. A second set of pictures was shown to the children and laid out in the grid. One set of numbers was placed beneath the two rows of pictures. The instructor counted to the correct quantity while placing the numbers. She then said, "There is another set of numbers that you can use to count the other pictures separately if you want," while gesturing. The cards with "more", "same" and "less" were shown and read to the child. The pictures in the first row were counted. The pictures in the second row were counted. "This row has 'more' and this row has 'less.' 'More.' 'Less'" The correct cards were placed at the end of the row. The lesson was repeated by removing the second set of pictures and laying out the third set of pictures. The researcher said, “You can use any two sets of the pictures.” A brief discussion with the children about what was needed for the rows to be the same, the other row to contain more, etc. then followed. The children were informed, “The pictures will change twice each week.” The material was correctly returned to the shelf.

The children in the experimental classroom were allowed to work with this material for one month before all children were given a post-assessment. The assessment was identical to the pre-assessment.

Data Analysis

Data was analyzed in several different ways. The primary variable of interest-conservation of number was analyzed using chi-square analysis and Pearson moment correlation analysis. Chi-square analysis was used to determine how much change occurred in this categorical variable between the initial assessment and the post- test. Pearson moment correlation analysis was used to verify the expected relationship between age and conservation of number.

Chapter 4: Results

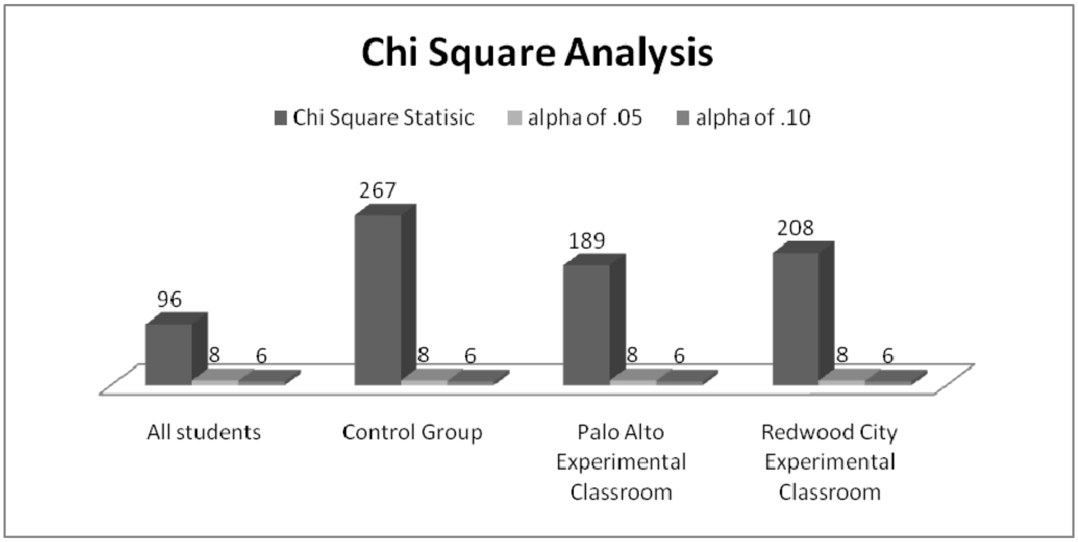
Conservation of Number

This research provided was designed to assess if specific curriculum changes affected the development of conservation of number in preschool children. There was one group of children which served as a developmental control group, and two groups of children which had the opportunity to work with an experimental material.

The chi-square square statistic is used to determine how much too categorical variables differ from each other. Each child's current conservation of number development was assessed at a particular level- 0, 1, 2, or 3 for both the pre-test and post-test. These children did not show either consistent or linear levels of conservation of number development. The assessed levels of many children either increased or decreased. The inconsistent level of development at this age makes it difficult to determine the effectiveness of the research project. Chi-square analysis was conducted to see if any changes between the pre-test and post-test were statistically significant. For all students, the control group, the experimental group in Palo Alto and the experimental group in Redwood City the changes shown in the children's levels of conservation of number were not statistically significant at either .05 or .10 probability levels.

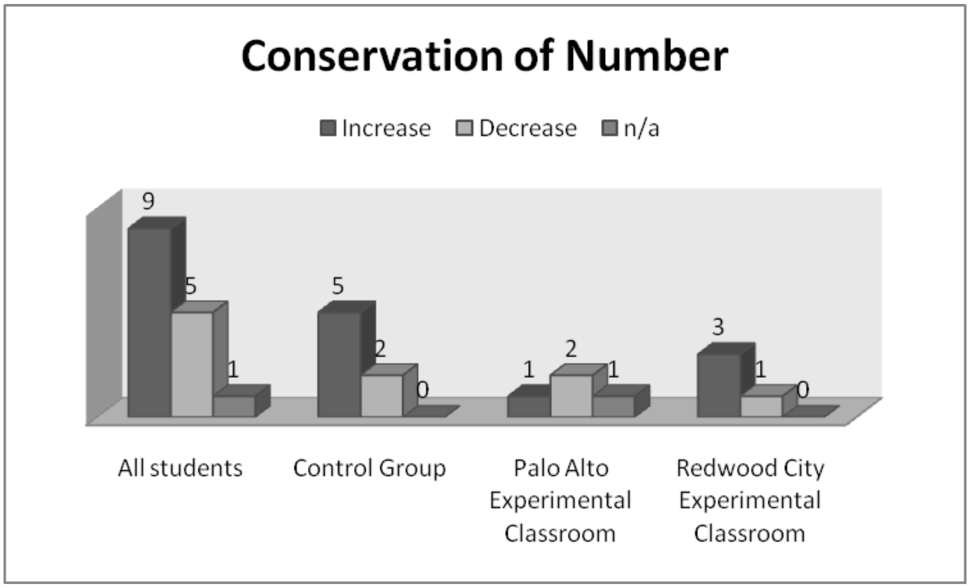
Probability tables (NIST - agent of U.S. Commerce Department's Technology Administration, 2006) indicate that value of 7.815 would need to be obtained for the chi-square statistic to be significant at a probability of .05. A value of 6.251 would need to be obtained at a probability of .10. The values obtained for the chi-square statistic are as follows: all students- 96.13; control group- 266.99; Palo Alto experimental group – 188.58; Redwood city experimental group- 208.07.

Figure 7: Chi-square Analysis



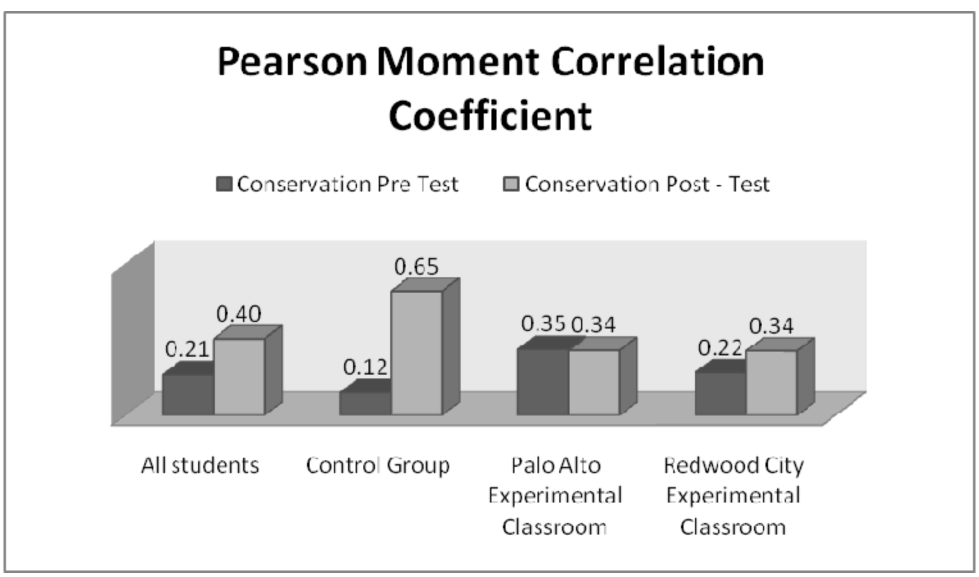
At the group level, the children demonstrated that conservation of number is not stable at this age. Both increases and decreases in the levels of conservation of number in individual children were noted in all groups of children. In the entire study population 9 children's scores increased while 5 children's scores decreased. In the control population 5 children's scores and 2 decreased. In the experimental population at Palo Alto, 1 increased and 2 decreased. In the experimental population at Redwood City 3 increased while 1 decreased. One student was not able to be assessed for conservation of number initially.

Figure 8: Changes in Conservation of Number



The Pearson Moment Correlation Coefficient was calculated for both the pre test and the post test scores to determine the correlation between age and conservation of number. The results showed a low to moderate correlation between age and the development of correspondence of number. Individual results affected the correlation coefficient.

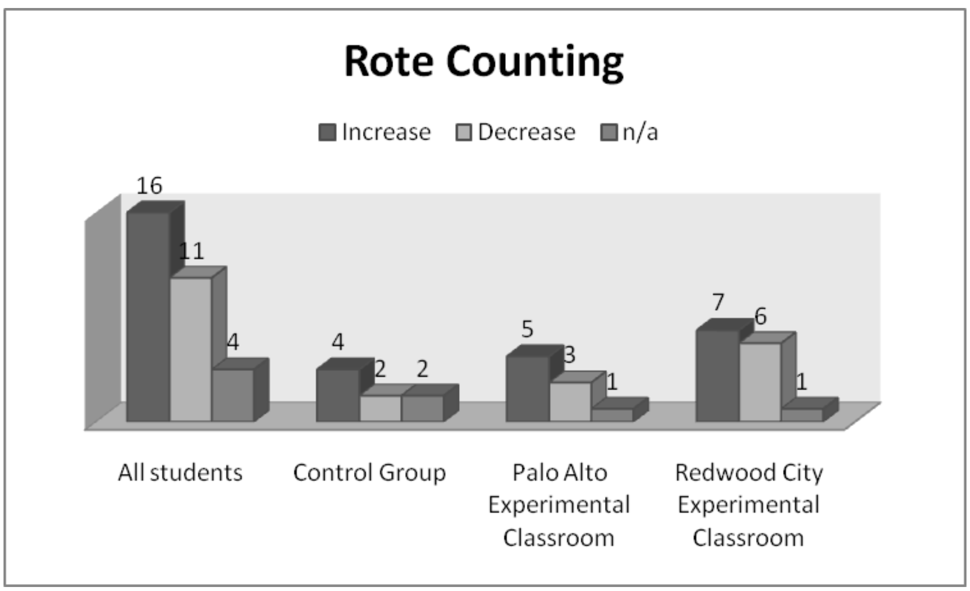
Figure 9: Pearson Moment Correlation Coefficient



Mathematical Assessment

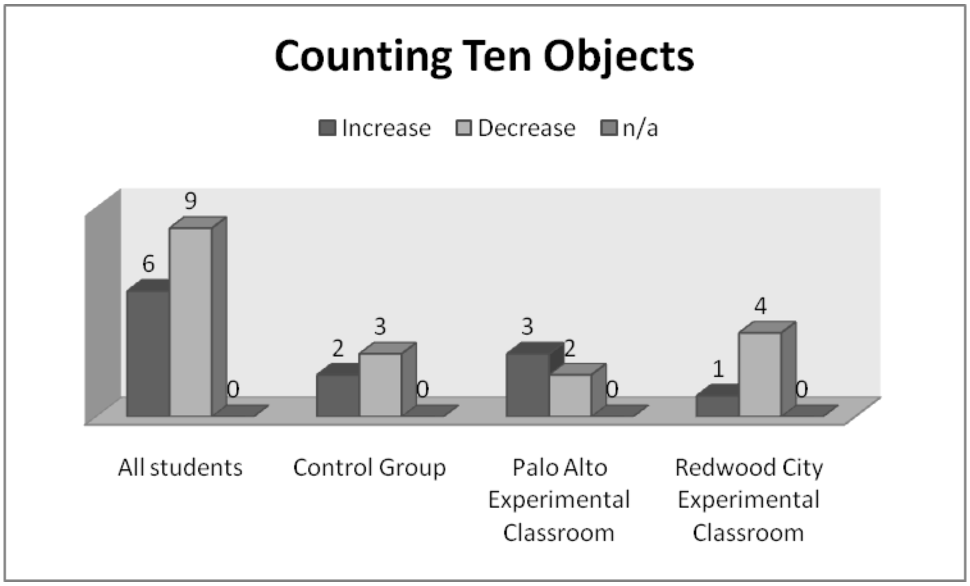
The individual results of the mathematical assessments also showed that children scores increased or decreased on all measures. When children were asked to demonstrate how high they could count sixteen increased their score while 11 decreased their score on the post assessment. (Control group – 4 increased, 2 decreased; Palo Alto Experimental 5 increased, 3 decreased; Redwood City Experimental 7 increased, 6 decreased)

Figure 10: Changes in Conservation of Rote Counting



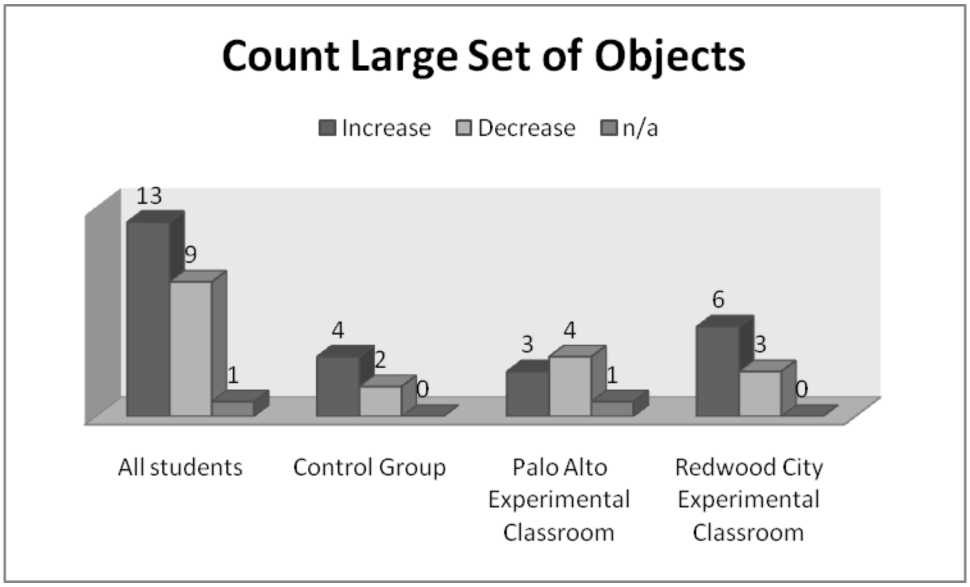
When asked to count out ten objects from a larger set of objects six additional students were able to do so at the post test while 9 students were unable to do so. (Control group - 2 increased, 3 decreased; Palo Alto Experimental - 3 increased, 2 decreased; Redwood - City Experimental 1 increased, 4 decreased)

Figure 11: Changes in Counting Ten Objects



Over the course of the experiment, 13 students showed an increase in their ability to count a large set of objects while 9 students demonstrated a decrease in this skill. (Control group - 4 increased, 2 decreased; Palo Alto Experimental - 3 increased, 4 decreased; Redwood City Experimental - 6 increased, 3 decreased)

Figure 12: Changes in Counting a Large Set of Objects



Conclusion

The results of this research are statistically insignificant. The research material appears to have had no affect on the development of conservation of number. The greatest increase was noted in the control group which also had the average age. Redwood City also showed increases in conservation of number and showed the lowest average age. This classroom reported actual usage of the material. The experimental classroom in Palo Alto showed more children that decreased their conservation of number skills, but actual usage was minimal.

Chapter 5: Discussion

This purpose of this research was to determine if certain curriculum changes would accelerate the development of conservation of number. The results are statistically insignificant for the entire sample and for each individual group. This result may be due to the numerous limitations of the study, especially actual usage in the classroom. Individual results, however, reflected some notable finding.

The researcher's observations in the classroom in Palo Alto indicated that the research material was rarely used. The children in this classroom were used to working in the class environment, but prior to the introduction of the research material had developed routines that generally led them to other areas of the classroom besides the mathematics area. Although new, the research material rarely attracted their attention.

The instructor in the classroom in Redwood City felt that the educational material was used frequently by several of the children in her classroom. They explored the material in unanticipated ways. This included working with three sets at a time, building larger sets out of two smaller sets and changing quantities in other ways. She felt that these children developed a much greater understanding of the concepts of "same," "more" and "less." The knowledge of these terms was not evaluated. She felt the material was very beneficial to the children in her classroom, and planned to continue its usage.

Individual Results- Conservation of Number

The results indicate that the development of conservation of number is not straightforward for children. The scores of 9 children increased while the scores of 5 children decreased. The majority of change documented was from one level to the next highest or lowest. One child, whose other math assessment scores were low on both the pre and post mathematical assessment, went from unable to form equivalent lines to the highest level of conservation of number.

Table 6: Individual Change in Conservation of Number

Student Group	Pre-Test Score	Post-Test Score	Change
Control	0	3	increase
Control	0	1	increase
Control	0	1	increase
Control	1	2	increase
Control	1	2	increase
Control	1	0	decrease
Control	1	0	decrease
Palo Alto- Experimental	0	1	increase
Palo Alto- Experimental	1	0	decrease
Palo Alto- Experimental	1	0	decrease
Redwood City- Experimental	0	1	increase
Redwood City- Experimental	0	1	increase
Redwood City- Experimental	0	1	increase
Redwood City- Experimental	2	0	decrease

Although the child's age corresponded to conservation of number development, the results were not straight forward. There was one three year old child that showed complete conservation of number at both the pre and post assessments. Several 4, 5, and 6 year olds were initially assessed at the lowest possible levels of conservation of number with little or no change. See Appendix D for the raw scores.

This research project examined conservation of number in relationship to curriculum changes. The short time period of the project, variation in differences in teaching style, wide developmental range and interests of the children made it difficult to determine the effectiveness of curriculum changes. This project documented the difficulties in determining if a young child has developed conservation of number. Dramatic improvements were shown, while the post assessment also showed children at a lower level in many cases. Neither age nor other math skills assessed would have been a consistent factor in the ability to predict conservation of number.

Donaldson (1978) indicated that children interpret questions in a way that is meaningful to them, irrespective of the language actually used. This was apparent with the conservation of number assessment and the mathematical assessment. When asked if the sets were “the same” several children closely examined each individual poker chip noting that the sides of each chip were different. This information may have been a consideration in their response. One child only scored a 1 on the conservation of number pre-assessment, but his response was extremely interesting. He had both matched and counted poker chips. After the transformation, he recounted them. Then he said, “I can count them and they are all the same, but they are not the same because they are longer.” Despite the standard adult definition of “same” and the research script asking about which row had more this child, clearly was defining “same” as identical in all aspects. His response raises the question, were other children treating same as identical or having other language issues?

Mathematical Assessment

The individual scores on all three components of the individual assessments also showed both increases and decreases from the pre-assessment to the post-assessment. It was apparent that the components of the assessment did not accurately assess the abilities of many of the children. Children were asked to show the research how high they could count. Some children would give a specific number- often relatively low (for example 12), and refuse to count past that number. However when counting a large set of objects they demonstrated rote counting skills substantially greater then on the rote counting assessment. Other children would say that was "easy" and wanted to move on to other parts of the assessment. When asked to count ten objects from a larger set, the most common error was to choose to count the entire set. Many of the children that had a negative score on the ability to count ten objects, actually accurately counted the entire set of fifteen objects that were present. The results of all aspects assessment made it impossible to look at relationships between the child's current conservation of number development without additional assessment.

Implications and Conclusion

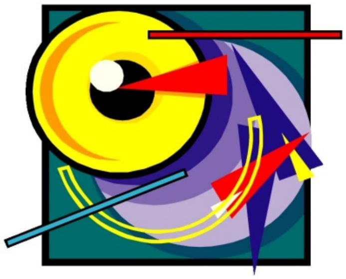
California mathematical standards for kindergarten students imply a level of mathematical understanding that can only be achieved when the child has achieved conservation of number. These standards include the following (CA State Board of Education, 2007).

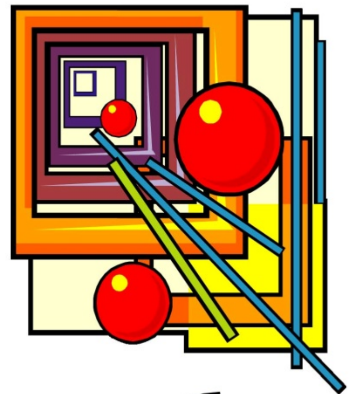
> 1.0 Students understand the relationship between numbers and quantities (i.e., that a set of objects has the same number of objects in different situations regardless of its position or arrangement):
>
> 1.1 Compare two or more sets of objects (up to ten objects in each group) and identify which set is equal to, more than, or less than the other. (CA State Board of Education, 2007)

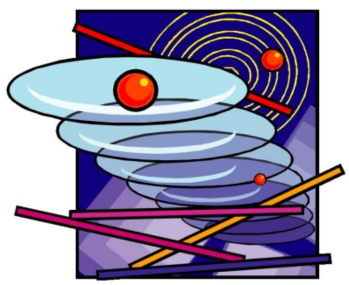
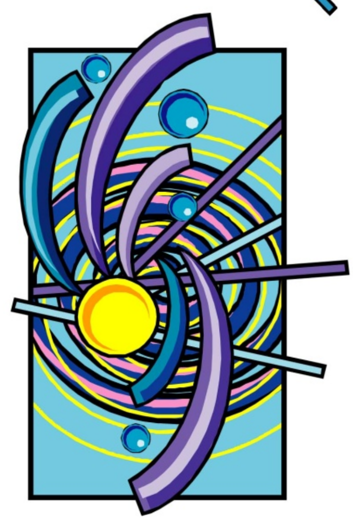
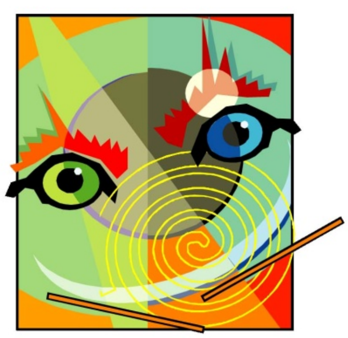
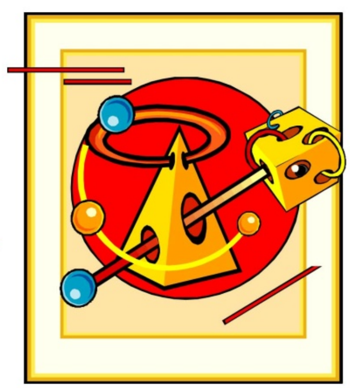
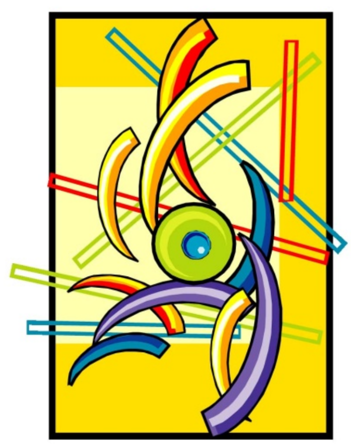
The relationship between quantity and arrangement is understood only once the concept of conservation of number has developed in the child. These standards make explicit the understanding that certain number skills are expected to be taught. The developmental nature of conservation of nature combined with increasing state standards makes research in this area relevant to the education of the young child.

The difficulty in assessing or predicting the development of conservation of number emphasizes the importance of providing a wide variety of experience with manipulatives and number. Early childhood curriculums should include concrete and symbolic mathematical experience. This includes counting, counting objects, manipulation of equivalent sets and unequal sets. This thesis documents the need for additional research into the unequal development of conservation of number. The original research question, “Does working with sets contribute to the development of conservation of number?” also needs additional research to be answered.

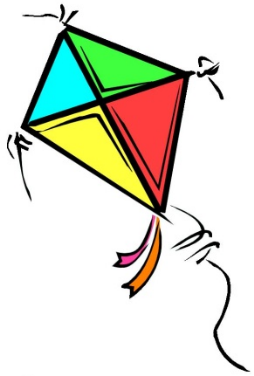
Appendix A – Pictures used in research

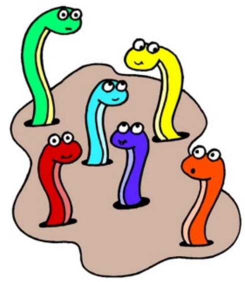




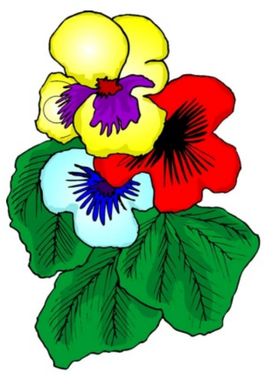






Appendix A – Pictures used in research












Appendix A – Pictures used in research

Appendix B – Student Assessment Form and Script

Student Assessment & Script

Date	
Time	
Student ID	
Researcher	**Tracy Crawford**

Rote Counting "Can you show me how high you can count?" (if needed "1, 2, 3..") (circle skipped numbers; Draw line after child stops.)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Counting Out Objects (1-10) "Can you count out 10 'suns' for me?"
Number counted out 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

# counted	**# indicated verbally**

Counting Set of Objects (max 30) "Can you count these for me for me?"
(subsequent sets) "Let's try these. Can you count these for me?"

30	**24**	**18**	**12**	**6**

Conservation of Number assessment (Script below)

Level 0	**Level 1- Non-conserving**	**Level 2- Transitional**	**Level 3- Conserving**
Child does not build equivalent rows.	**Child does not believe equivalence is maintained w/ perceptual transformation.**	**Equivalence maintained w/ perceptual transformation; Lacks justification. Indecision in judgment.**	**Equivalence maintained and provides justification.**
Justification -			

Conservation of Number script
Place out 8 red poker chips.
"Can you put out the same number of blue poker chips?"
Spread out red poker chips
"Are there the same amount of red chips as blue chips or are there more red or blue chips?"
"Are there still the same amount of red chips as blue chips or are there more red or blue chips?"
"Can you tell me why?"

Appendix C – Educational Material Presentation

1. Unroll rug and get materials from their position of the shelf.
2. Lay out board.
3. Show the children first set of pictures; Lay in grid.
4. Show the children second set of pictures; Lay in grid.
5. Show the children first set of numbers and count while placing numbers beneath pictures.
6. "There is another set of numbers that you can use to count the other pictures separately if you want."
7. Show the cards with "more," "same" and "less" to children and read cards to children.
8. Place correct cart at the end of each row. Repeat words while gesturing to each row.
9. Remove second set of pictures; Demonstrate again with first and third set.
10. "You can use any two sets of the pictures."
11. "The pictures will change twice each week

Appendix D – Conservation of Number Scores with Ages

Sorted by Pretest, Age, and School

Student Group	**Male / Female**	**Age**	**Pre-Test**	**Post-Test**	**Change**
Control	Male	5.1	3	3	
Control	Male	4.3	1	0	decrease
Control	Male	4.4	1	2	increase
Control	Female	4.6	1	1	
Control	Female	4.6	1	0	decrease
Control	Female	5.0	1	1	
Control	Male	5.1	1	2	increase
Control	Male	5.2	1	1	
Control	Male	5.2	1	1	
Control	Male	5.2	1	1	
Control	Female	5.3	1	1	
Control	Male	4.1	0	0	
Control	Female	4.2	0	0	
Control	Female	4.5	0	1	increase
Control	Male	4.6	0	0	
Control	Male	4.8	0	0	
Control	Female	5.4	0	1	increase
Control	Male	6.2	0	3	increase
Palo Alto-Experimental	Male	4.3	n/a	1	n/a
Palo Alto-Experimental	Male	5.1	3	3	
Palo Alto-Experimental	Female	5.5	3	3	
Palo Alto-Experimental	Female	3.9	1	0	decrease

Palo Alto-Experimental	Female	4.4	1	1	
Palo Alto-Experimental	Female	4.5	1	1	
Palo Alto-Experimental	Male	4.6	1	1	
Palo Alto-Experimental	Male	5.1	1	1	
Palo Alto-Experimental	Female	5.2	1	1	
Palo Alto-Experimental	Female	5.3	1	1	
Palo Alto-Experimental	Female	5.4	1	1	
Palo Alto-Experimental	Female	5.7	1	0	decrease
Palo Alto-Experimental	Male	3.7	0	0	
Palo Alto-Experimental	Female	4.0	0	0	
Palo Alto-Experimental	Male	4.1	0	0	
Palo Alto-Experimental	Female	4.2	0	0	
Palo Alto-Experimental	Male	5.0	0	0	
Palo Alto-Experimental	Male	5.0	0	1	increase
Palo Alto-Experimental	Male	5.5	0	0	
Palo Alto-Experimental	Female	5.5	0	0	
Redwood City-Experimental	Female	3.8	3	3	
Redwood City-Experimental	Male	5.0	3	3	
Redwood City-Experimental	Male	3.7	2	0	decrease
Redwood City-Experimental	Male	5.4	2	2	
Redwood City-Experimental	Female	4.4	1	1	
Redwood City-Experimental	Male	4.5	1	1	

Redwood City-Experimental	Female	4.6	1	1	
Redwood City-Experimental	Female	4.6	1	1	
Redwood City-Experimental	Male	5.1	1	1	
Redwood City-Experimental	Male	5.3	1	1	
Redwood City-Experimental	Male	3.4	0	0	
Redwood City-Experimental	Male	3.5	0	0	
Redwood City-Experimental	Male	3.8	0	0	
Redwood City-Experimental	Female	3.9	0	1	increase
Redwood City-Experimental	Female	4.0	0	0	
Redwood City-Experimental	Male	4.2	0	1	increase
Redwood City-Experimental	Male	4.6	0	0	
Redwood City-Experimental	Male	4.6	0	1	increase
Redwood City-Experimental	Female	5.3	0	0	

References

Association of Bay Area Governments. (2003, Oct 1). Bay Area Census Data-Redwood City. Retrieved August 23, 2008, from Association of Bay Area Governments: http://www.bayareacensus.ca.gov/cities/RedwoodCity.htm

Association Montessori Internationale. (2008). Montessori Approach. Retrieved June 21, 2008, from Association Montessori Internationale: http://www.montessori-ami.org/

Blitzer, C. (n.d.). *About neighborhoods*. Retrieved March 25, 2008, from Palo Alto online: http://www.paloaltoonline.com/neighborhoods/

Brainerd, C. (1977). Cognitive development and concept learning: An intrepretative review. *Psychological Bulletin, 84*, 919-939.

Bryant, P. (1972). The understanding of invariance by very young children. *Canadian Journal of Psychology*, 78-96.

Bryant, P., & Trabasso, T. (1971). Transitive inferences and memory in young children. *Nature, 232*, 456-458.

CA State Board of Education. (2007, June 25). *CA State Board of Education*. Retrieved April 4, 2008, from Kindergarten mathematical content standards: http://www.cde.ca.gov/be/st/ss/mthkindergarten.asp

Caddell, D. (1998). *Numeracy in the early years: What the research tells us.* Dundee, Scotland: Learning and Teaching Scotland.

Chattin-McNichols, J. (1998). *The Montessori controversy.* Albany, NY: Delmar publishers.

Coldwell Banker (2007, November 6). *Press release*. Retrieved March 25, 2008, from Coldwell banker: http://hpci.coldwellbanker.com/hpci_press.aspx

Crain, W. (2005). *Theories of development: Concepts and applications.* Upper Saddle River, NJ: Pearson Prentice Hall.

Daoust, C. (2004, Spring). An examination of implementation practices in Montessori early childhood education. *Doctoral dissertation, University of California, Berkeley* .

Deal, T. N. (1967). *Effects of a structured program of preschool mathematics on cognitive behavior.* Athens, GA: University of Georgia.

DeVries, R. (2002, June). What does research on constructivist education tell us about effective schooling? *The Iowa Academy of Education Occasional Research Paper #5* . Des Moines, Iowa: FINE (First in the Nation in Education) Foundation.

DeVries, R., & Goncu, A. (1968). Interpersonal relations in four-year-old dyads from constructivist and Montessori programs. *Early Child Development and Care*, 11-27.

Donaldson, M. (1978). *Children's minds.* New York: W.W. Norton & Company.

Enotes.com. (2008). Retrieved June 21, 2008, from Famous quotes-The principle goal of Education: http://www.enotes.com/famous-quotes/the-principle-goal-of-education-in-the-schools

Field, D. (1981). Can preschool children really learn to conserve? *Child Development*, *52*(1), 326-334.

Fuson, K. C., Pergament, G. G., Lyons, B. G., & Hall, J. W. (1985). Children's conformity to the cardinality rule as a function of set size and counting accuracy. *Child Development, 56*(6), 1429-1436.

Fuson, K. C., Secada, W. G., & Hall, J. W. (1983). Matching, counting, and conservation of numerical equivalence. *Child Development, 54*(1), 91-97.

Gelman, R. (1969). Conservation acquisition: A problem of learning to attend to relevant attributes. Journal of Experimental Child Psychology, 7(2), 167-187.

Gelman, R. (1972). Logical capacity of very young children: Number invariance rules. *Child Development, 43*(1), 75-90.

Gelman, R. (1979). Preschool thought. *American Psychologist, 34*(10), 900-905.

Gelman, R., & Gallistel, C. (1986). *The child's understanding of number*. Cambridge, MA: Harvard University Press.

Halford, G. S., & Boyle, F. M. (1985). Do young children understand conservation of number? *Child Development, 56*(1), 165-176.

Henry, D. (1976). Interrelationships among attentional preferences, cardinal-ordinal ability, and conservation of number. *Child Development, 47*(3), 750-758.

International Montessori Index. (2008, May 15). FAQ's (Frequently Asked Questions). Retrieved June 21, 2008, from Montessori: The International Montessori Index: http://www.montessori.edu/FAQ.html#QUESTIONS

Kamii, C. (1982, Jan 14). Constructivist education: A direction for the 21st century. *Paper presented at a lecture given in celebration of the 10th anniversary of Circle Children's Center*. Chicaco, Illinois.

Kamii, C. (1988). Number in preschool and kindergarten. Washington, DC: National Association for the Education of Young Children.

Kamii, C. (2000). *Young children reinvent aritmetic*. New York: Teachers College Press.

Kamii, C. (2001). Young children's representation of groups of objects: The relationship between abstraction and representation. *In Proceedings of the Annual Meeting of the North American chapter of the International Group for the Psychology of Mathematics Education*, (pp. 207-212). Snowbird, UT: University of Alabama at Birmingham.

Katz, H., & Beilin, H. (1976). A test of Bryant's claims concerning the young child's understanding of quantitative invariance. *Child Development, 47*(3), 877-880.

Kaufman, E., Lord, M. W., Reese, T. W., & Volkmann, J. (1949). The discrimination of visual number. The American Journal of Psychology, 62(4), 498-525.

Lemoyne, G., & Favreau, M. (1981). Piaget's concept of number development: Its relevance to mathematics learning. *Journal for Research in Mathematics Education*, *12*(3), 179-196.

Lillard, A., & Else-Quest, N. (2006). Evaluating Montessori education. *Science*, 313, 1893-1894.

Macnamara, J. (1975). A Note on Piaget and number. *Child Development*, *46*(2), 424-429.

McClintic, S. (1988). Conservation - a meaningful gauge for assessment. *The Arithmetic teacher*, 12-14.

Mehler, J., & Bever, T. G. (1967). Cognitive capacity of very young children. *Science, 153*, 141-142.

Miller, P. H., & West, R. F. (1976). Perceptual supports for one-to-one correspondence in the conservation of number. Journal of Experimental Child Psychology, 21(3), 417-424.

Miller, P., Heldmeyer, K., & Miller, S. (1975). Facilitation of conseration of number in young children. *Developmental Psychology, 11*(2), 1-17.

Mooney, C. G. (2000). *An introduction to Dewey, Montessori, Erikson, Piaget, and Vygotsy*. St. Paul, MN: Redleaf Press.

NIST - agent of U.S. Commerce Department's Technology Administration. (2006, July 18). Critical Values of the Chi-Square Distribution. Retrieved August 28, 2008, from Engineering Statistics Handbook: http://www.itl.nist.gov/div898/handbook/eda/section3/eda3674.htm

Piaget, J. (1965). *The Child's conception of number*. New York: W. W. Norton & Company.

Price, D. A. (2004, July). Is it time to let go of conservation of number? Retrieved August 3, 2008, from The Electronic Library of Mathematics: Mathematical collections and conference proceedings: http://www.emis.de/proceedings/PME28/SO/SO079_Price.pdf

Rose, S. A. (1973). Acquiescence and conservation. *Child Development, 44*(4), 811-814.

Rose, S. A., & Blank, M. (1974). The potency of context in children's cognition: An illustration through conservation. *Child Development , 45*(2), 499-502.

Rothenberg, B., & Courtney, R. (1969, October). Merrill-Palmer quarterly. *A Developmental study of nonconservation choices in young children*, 363-373.

Rothenberg, B., & Orost, J. H. (1969). The training of conservation of number in young children. *Child Development*, *40*(3), 707-726.

Samuel, J., & Bryant, P. (1984). Asking only one question in the conservation experiment. *Journal of Child Psychology and Psychiatry*, *25*(2), 315-318.

Saxe, G. B. (1979). Developmental relations between notational counting and number conservation. *Child Development*, *50*(1), 180-187.

Schneck, B., & Canaday, H. (1974). A failure to teach very young children to conserve number. Virginia State College. Petersburg, VA: Cooperative Extension Services.

Siegel, L. S., & Goldstein, A. G. (1969). Conservation of number in young children: Recency versus relational response strategies. Developmental Psychology, 1(2), 128-130.

Sophian, C. (1995). Representation and reasoning in early numerical development: Counting, conservation, and comparisons between sets. *Child Development*, *66*(2), 559-577.

Starkey, P. (1981). Young children's performance in number conservation tasks: Evidence for a hierarchy of stagies. *The Journal of Genetic Psychology*, *138*, 103-110.

Thompson, D. (1995). Preschool math and the didactic materials. *Montessori Life*, 20-23.

Wallach, L., & Sprott, R. L. (1964). Inducing number conservation in children. *Child Development*, *35*(4), 1057-1071.

Whiteman, M., & Peisach, E. (1970). Perceptual and sensormotor supports for conservation tasks. *Developmental Psychology*, *2*(2), 247-256.

Willoughby, R., & Trachy, S. (1971). Conservation of number in very young children: A failure to replicate Mehler and Bever. *Merrill-Palmer Quarterly of Behavior and Development*, *17*(3), 205-209.

Winer, G. (1974). Conservation of different quantities among preschool children. *Child Development* , *45*, 839-842.

Yussen, S., Matthews, S., & Knight, J. (1980). Performance of Montessori and traditionally schooooled nursery children on social cognitive tasks and memory problems. *Contemporary Educational Psychology, 5*,124-137.

Zhou, X. (2002). Preschool children's use of counting to compare two sets in cardinal situations. *Early Child Development and Care, 172*(2), 99-111.

Zimiles, H. (1965). *The Development of differentiation and conservation of number.* New York: Bank Street College of Education.

Zusne, L., & Jones, W. H. (1989). Anomalistic Psychology: A Study of Magical Thinking. Mahwah, NJ: Lawrence Erlbaum.